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Soft collisions in relativistic runaway electron avalanches

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Abstract
This paper reports the first application of the relativistic binary-encounter-Bethe (RBEB) electron impact ionization model for studies of relativistic runaway electron avalanches (RREA) phenomenon at different pressures in air, which is believed to be the root cause of the hard x-rays and terrestrial gamma-ray flashes observed in the Earth’s atmosphere in association with lightning activity. The model allows robust and accurate description of ionization over a wide range of energies (from the ionization threshold to megaelectronvols), that is especially important for studies of thermal runaway electrons. A direct comparison between RREA rates obtained using classic Møller and the new RBEB differential ionization cross sections demonstrates that the dipole interaction between primary electrons and K-shell electrons of oxygen and nitrogen has an impact on the rates for relatively low applied electric fields comparable to or higher than 20 kV cm\(^{-1}\) at ground pressure. Implications of non-similarity of the runaway process developing at different altitudes due to the Swann–Fermi density effect are discussed.

1. Introduction

The enhancement of electric fields around tips of streamers is one of the unique naturally occurring circumstances in which fields \(\sim 10E_b\), where \(E_b\) is the conventional breakdown threshold field defined by the equality of the ionization and dissociative attachment coefficients in air, can be dynamically produced and sustained for relatively extended periods of time. The ability of these streamer tip fields to generate runaway electrons was identified and discussed in the literature over two decades (Babich (1982) and references therein), and a new insight on this phenomenon has been given by recent laboratory experiments (Dwyer et al 2008, Nguyen et al 2008, 2010, Rahman et al 2008). Moreover, it has been proposed that with total potential differences on the order of tens of megavolts available in streamer zones of lightning leaders, during a highly transient negative corona flash stage of the development of negative stepped leader, electrons with energies 2–8 keV ejected from streamer tips near the leader head can be further accelerated to energies of hundreds of kiloelectronvolts and possibly to several tens of megaelectronvolts, depending on the particular magnitude of the leader head potential (Moss et al 2006). It has been proposed that these energetic electrons may be responsible (through the bremsstrahlung process) for the generation of hard x-rays observed from ground and satellites (e.g. Carlson et al (2009), Cummer et al (2005), Dwyer et al (2005), Fishman et al (1994), Inan et al (1996), Moore et al (2001), Smith et al (2005) and references therein). The current theories of transient luminous events occurring above cloud tops and termed blue and gigantic jets generally favour a phenomenological link between jet discharges and streamer zones of lightning leaders (Krehbiel et al (2008) and references therein) and it has been suggested that the thermal runaway electron process operating in leaders may contribute to the production of terrestrial gamma-ray flashes from the jet discharges (Moss et al 2006). This mechanism has been further supported by recent analysis indicating that peak fields in these discharges derived from spectrophotometric measurements had been significantly underestimated (Celestin and Pasko 2010).

The need for an accurate description of the electron runaway phenomena in air involving electron energy distributions with low initial energy (\(\sim 1\) eV) for a broad range of applied electric fields and describing coupling between high and low energy electron processes over many decades of electron energy (from sub-electronvolts to several megaelectronvolts) is an important task of the current
research actively pursued by several research groups (e.g. Channion and Neubert (2008, 2010), Colman et al (2010)).

In the study of the creation of runaway electrons with energies higher than 1 keV, usually only hard collisions (free projectile electron and free target electron) are considered through the Møller cross section (e.g. Dwyer and Smith (2005), Roussel-Dupré et al (1994)). This assumption is accurate if the energy of the secondary electron is much higher than its binding energy with the nucleus. This paper presents the first application of the relativistic binary-encounter-Bethe (RBEB) model (Kim et al 2000), which provides a robust and accurate description of ionization over a wide range of energies (from the ionization threshold to megaelectronvolts), to studies of relativistic runaway electron avalanche (RREA) phenomena at different pressures in air. We demonstrate that soft ionizing collisions between electrons and molecules (when target electrons cannot be considered as free particles) play a direct role in the RREAs developing under applied fields above $\sim 20 \text{ kV cm}^{-1}$ in air at ground pressure. We also discuss non-similarity of RREAs developing at different altitudes due to the Swann–Fermi density effect.

2. Model formulation

2.1. Relativistic binary-encounter-Bethe model

The RBEB model (Kim et al 2000) is the relativistic extension of the binary-encounter-Bethe (BEB) model developed by Kim and Rudd (1994). The BEB model, applied to neutral molecules by Hwang et al (1996), gives very good results at low energy and is used to compute molecular databases provided by the National Institute of Standards and Technology (http://physics.nist.gov/PhysRefData/Ionization/molTable.html). The RBEB model is based on the combination of the free–free Møller cross section and the leading dipole part of the relativistic Bethe cross section (e.g. Inokuti (1971)). It results in an analytical representation of singly differential ionization cross section, which continuously covers a range of energy from the ionization threshold to the megaelectronvolt range. This continuous coverage over broad energy range makes it of first interest for the simulation of RREA and thermal runaway processes. Moreover, the RBEB model does not need any adjustable parameters and has been shown to be in very good agreement with the experiments (e.g. Kim et al (2000), Santos et al (2003)). This model only requires the average orbital kinetic energy and the binding energy of the target electron. These quantities are readily available in the literature (e.g. Hwang et al (1996), Santos et al (2003)).

In this work, we have computed the singly differential ionization cross section for O$_2$ and N$_2$ from the RBEB model for each molecular orbital and we summed them in the proportion of air (20% O$_2$ and 80% N$_2$). As will be shown in section 4, the K-shells are of particular importance in the calculations of this work, and we emphasize that the RBEB model provides accurate results for K-shells of light atoms such as N and O (Santos et al 2003).

![Figure 1](image-url) Comparison between the dynamic friction forces defined by the Bethe formula and computed using the RBEB model with respect to the energy.

2.2. Monte Carlo model

The Monte Carlo model developed in this work simulates the propagation and creation of electrons with kinetic energies between $E_{\text{min}} = 2 \text{ keV}$ and 1 GeV, under the influence of constant applied electric field of magnitude $E$. The model closely follows the development presented by Lehtinen et al (1999). The present version of the model does not take into account ambient magnetic field. We consider homogeneous electric fields. The model is three-dimensional (3D) axisymmetric about the electric field in the momentum space and zero-dimensional (0D) in the configuration space.

While gaining energy from the electric field $E$, the electrons are losing energy from collisions with the molecules of the gas. The quantity associated with this process is the dynamic friction force $F_D$, which characterizes the average energy loss of an electron per unit of length. For energetic electrons, $F_D$ can be defined by the Bethe formula (Bethe and Ashkin 1953, p 254, equation (52)), which takes into account hard (free-free) and soft (free-dipole) collisions. This function has a minimum at the electron kinetic energy $\mathcal{E} \simeq 1.2 \text{ MeV}$ (see figure 1). From this minimum we define the runaway threshold electric field $E_r/N \simeq 8 \text{ Td}$ ($1 \text{ Td} = 10^{-21} \text{ V m}^2$), where $N$ is the air density, above which an electron could gain more energy from the field than it loses due to collisions. Figure 1 shows $F_D$ calculated using the RBEB model and the Bethe formula applied to air at atmospheric density at the ground level $N_0 = 2.688 \times 10^{27} \text{ m}^{-3}$. We see that both models give close results. In the Monte Carlo simulations presented below, we tested that the substitution of $F_D$ given by RBEB with the Bethe formula does not lead to noticeable differences in results.

An electron gains energy through the relativistic equation of motion under the influence of the field $E$ and is slowed down through $F_D$:

$$\frac{dp}{dt} = q_e E - F_D \frac{p}{p},$$

where $p = \gamma m v$ is the relativistic momentum, $\gamma$, $m$ and $v$ being respectively, the Lorentz factor, the mass and the velocity
of the electron, and $q_e$ is the charge of the electron. The energy exchanges due to ionizing collisions creating secondary electrons with the energy $E_s > E_{\text{min}}$ are explicitly taken into account in our model. Therefore, the ionization processes leading to secondaries $E_s > E_{\text{min}}$ are subtracted from $F_p$ before applying this force to the electrons (see Lehtinen et al 1999, equation (5)).

In the simulation, the time step is chosen so that one primary electron can hardly produce more than one secondary electron with an energy higher than 2 keV within one time step. However, during this time step, electrons experience a lot of elastic collisions. Therefore, the angular diffusion of electrons is mainly considered through the process of multiple scattering of electrons from elastic collisions. The method of computation is set as prescribed in Lehtinen et al (1999) by considering small angles of deviations $\Delta \Theta$ and the equation:

$$\frac{d(\Theta^2)}{dt} = \frac{4\pi N Z^2 e^4}{u^3 \gamma^2} \ln \left( \frac{164.7}{Z_m^{1/3}} \frac{p}{mc} \right),$$

where $c$ is the speed of light in vacuum, $r_c$ is the classical electron radius and $Z_m \approx 14.5$ is the mean molecular charge for air. The momentum of the primary and secondary electrons after an ionizing collision is determined by the energy–momentum conservation law, that is the angles $\theta_p$ and $\theta_s$ between the momentum of the primary electron before collision and the primary and secondary electrons after ionizing collision, respectively, are defined by the formulae:

$$\cos \theta_p = \frac{(E_p - E_s)(E_p + 2mc^2)}{E_p(E_p - E_s + 2mc^2)},$$

$$\cos \theta_s = \frac{E_s(E_p + 2mc^2)}{(E_p + 2mc^2)},$$

where $E_p$ and $E_s$ are the energy of primary and secondary electrons, respectively.

The simulation starts with 5000 electrons launched at 1 MeV in the antiparallel direction of the electric field. In order to keep track of primary and secondary electrons without increasing the computation times much, we have developed a 2-bins remapping technique (Kunhardt and Tzeng 1986, Moss et al 2006). This technique reduces the number of particles effectively taken into account in the computations by merging them into super-particles, such that each of the super-particles actually represents many particles. First, the energy spectrum of electrons is separated into two groups. Then, the remapping method is applied on the lowest energy group, such that the information on high-energy particles is not degraded (Moss et al 2006, section 2.6). A representation of the results given by such a method is illustrated in figure 2.

The probability of creation of a secondary electron having the kinetic energy $E_s > E_{\text{min}}$ by electron impact with a primary electron with energy $E_p$ during the timestep $\Delta t$ is given by $P(E_p, E_s > E_{\text{min}}) = N v \Delta t \sigma(E_p, E_s > E_{\text{min}})$ (Lehtinen et al 1999), where $N$ is the gas density, $v$ is the velocity of the primary electron and $\sigma(E_p, E_s > E_{\text{min}})$ is the cross section of this event that is defined as

$$\sigma(E_p, E_s > E_{\text{min}}) = \int_{E_{\text{min}}}^{E_s} \frac{d\sigma_{\text{ion}}}{dE_s}(E_p, E_s) \, dE_s,$$

where $d\sigma_{\text{ion}}/dE_s$ is the singly differential cross section of ionization. In this work $\sigma(E_p, E_s > E_{\text{min}})$ has been defined either by Möller differential cross section with a mean molecular charge $Z_m = 14.5$ or by the RBEB model applied to O2 and N2. The comparison of $\sigma(E_p, E_s > E_{\text{min}})$ given by these models is shown in figure 2 (see section 4).

The energy of the secondary electron is determined through the generation of a random number $R_s$ in the interval [0, 1] and the numerical inversion of the singly differential cross section given either by Möller or RBEB models (e.g. see Lehtinen et al (1999), equation (17)), that is by solving numerically the following equation for $E_s$:

$$R_s = \frac{1}{\sigma(E_p, E_s > E_{\text{min}})} \int_{E_{\text{min}}}^{E_s} \frac{d\sigma_{\text{ion}}}{dE_s}(E_p, E_s) \, dE_s.$$

The critical runaway energy $E_{\text{run}}$ above which an electron is considered to be runaway is computed for each applied electric field studied by comparison between the applied electric field, the dynamic friction force, and the average cosine of angles between momentum of electrons and the direction of the applied electric field, as fully described in Lehtinen et al (1999). As shown in figure 1 the dynamic friction forces given by the Bethe formula and RBEB cross sections are in good agreement (see table 1), therefore the values of $E_{\text{run}}$ given by both of these approaches give very close results. Indeed, the variations in values of $E_{\text{run}}$ between the two models shown in table 1 do not lead to noticeable differences in the results of simulation.

![Figure 2](image-url)
The simulation of photons and positrons and therefore related feedback effects (e.g. Babich et al (2005), Dwyer (2003)) are not included in the present study.

3. Results

In the literature, results on RREA rates are often provided with respect to the relativistic runaway over-voltage \( \delta = E / E_a = E (V m^{-1}) / 215 \times 10^3 \) at the ground level air density \( N_0 = 2.688 \times 10^{25} m^{-3} \), where \( E \) is the applied electric field (e.g. Babich et al (2001), Lehtinen et al (1999)). The computations have been realized for applied electric fields corresponding to \( \delta = 2, 5, 8, 10, 12, 15 \) and 20. For each applied electric field, we have launched 10 simulations during a time of 1 \( \mu \)s in order to accurately evaluate the statistical errors in the results, presented by the error bars in figure 5.

Figure 3 shows a representative distribution of electrons in the momentum space at \( t = 64.6 \) ns for an applied electric field with magnitude corresponding to \( \delta = 5 \). In this figure, superparticles are represented by points with areas proportional to the number of actual electrons they represent because of the remapping technique. As discussed above, since we want to keep a high resolution for the particles with the highest energies, we clearly see that only particles with lowest energies are affected by the remapping technique. The quantities \( p_z \) and \( p_t \) in figure 3 are the momentum components along and perpendicular to the direction of the electric field, respectively. Figure 3 also demonstrates the highly directional motion of electrons along the direction of the electric field.

From the results of the Monte Carlo simulations we can compute the electron energy distribution function (EEDF). As an example, figure 4 shows the EEDF normalized to 1 for energies higher than \( E_{min} = 2 \) keV, for an applied electric field corresponding to \( \delta = 10 \) and using the RBEB model for ionization. That is,

\[
\int_{E>2\text{keV}} f(E) \, dE = 1,
\]

where \( f(E) \) is the EEDF. The characteristic features of this distribution are in good agreement with distributions shown in Lehtinen et al (1999, Figure 5).

The number of runaway electrons \( N_{run} \) increases in time and can be fitted by the relation \( N_{run} = 5000 \exp(\gamma t) \), where 5000 is the initial number of electrons and \( \gamma \) is the RREA rate. Figure 5 presents the RREA rates obtained in this study with respect to \( \delta \). In figure 5, rates are presented in terms of \( \tilde{\gamma} = \gamma (\text{ns}^{-1}) \times 171.53 \) versus the relativistic runaway over-voltage \( \delta = E (V m^{-1}) / 215 \times 10^3 \), for \( \delta = 8, 10, 12, 15 \) and 20. Rates for lower \( \delta \) given by both models have been observed to be identical.

The RREA rates computed using the Møller cross section are in good agreement with those documented in Lehtinen et al (1999), which validates the numerical techniques employed in this work. These rates are also confirmed by other research groups (e.g. Babich et al (2001), Coleman and Dwyer (2006)).

The rates obtained using the RBEB model are similar at low electric field to those obtained with the Møller model. Indeed, the rates have been found to be identical between
4. Discussion

We have verified that the deviation obtained in RREA rates (figure 5) mainly comes from the discrepancies between cross sections shown in figure 2. In fact, the relative difference between the cross sections represented in figure 2 is as high as ~15% for $\varepsilon_F \approx 1$ MeV and ~35% for $\varepsilon_F \approx 100$ MeV. The increase in the cross section given by the RBEB model for $\varepsilon_F \geq 1$ MeV is the high-energy part ($\varepsilon_F > 2$ keV) of an effect called the relativistic rise in cross sections and is due to the dipole interaction (e.g. Santos et al (2003)) taken into account in the RBEB model, and not included in the Møller cross section that only characterizes hard collisions (free electron colliding with a free electron at rest). The Møller cross section gives accurate results if $\varepsilon_F \gg B$, where $B$ is the binding energy of the target electron. Increasing the applied electric fields results in decreasing $\varepsilon_{\text{run}}$. For example, for $\delta = 15$ we find $\varepsilon_{\text{run}} \approx 24$ keV, whereas the binding energies of the K-shell electrons of O and N are $\approx 0.54$ keV and $\approx 0.41$ keV, respectively (e.g. Santos et al (2003)). Although the Møller cross section has been considered as sufficiently accurate, figure 5 shows that the dipole-type interaction plays a non-negligible role in the ionization process leading to secondary electrons created with energies of a few tens of kiloelectronvolts.

Indeed, artificially setting the binding energies of K-shell electrons to energies typical for L-shell electrons (tens of eVs) in the RBEB formulation makes the cross sections presented in figure 2 match, since in this case only hard collisions ($\varepsilon_F \gg B$) are taken into account, and thus RREA rates get very close to those computed using the Møller cross section.

The results presented in this paper on RREA rates have been obtained at the density of air at ground level $N_0 = 2.688 \times 10^{25}$ m$^{-3}$ and are scalable proportionally to the density of air allowing to find related rates for the same reduced electric fields $E/N$ at any altitude of interest. The passage of high-energy electrons through a medium results in an effect of polarization of atoms known as the Swann–Fermi density effect (Fermi 1940), or simply density effect, which can be introduced directly in cross sections (e.g. ICRU Report 37 (1984), Scofield (1978)), and is related to the density of the gas. Thus, the density effect introduces a non-similarity between RREAs propagating at different altitudes. In order to present scalable results with altitude, we have verified that the deviation obtained for $E/N$ the use of Møller cross section cannot be used for ionization processes leading to low energy secondary electrons (see discussion in Roussel-Duprê et al (1994)). The RBEB model allows for computing accurate ionization cross sections seamlessly over a very wide range of energy (from ionization threshold to megaelectronvolts) for both primary and secondary electrons, that is especially important for studies of thermal electron runaway phenomena (e.g. Chanrion and Neubert (2010), Colman et al (2010)). Since the RBEB formulation is analytical, it is directly implementable in Boltzmann or Monte Carlo codes, and since it does not require any adjustable parameters it is straightforward to apply it for different gases, for example, in order to simulate relativistic breakdown in planetary atmospheres (e.g. Dwyer (2007), Roussel-Duprê et al (2008)).

5. Summary and concluding remarks

Principal results of this work can be summarized as follows:

(i) We have reported the first application of the RBEB model to studies of RREA phenomena in air. It is well known that Møller cross section cannot be used for ionization processes leading to low energy secondary electrons (see discussion in Roussel-Duprê et al (1994)). The RBEB model allows for computing accurate ionization cross sections seamlessly over a very wide range of energy (from ionization threshold to megaelectronvolts) for both primary and secondary electrons, that is especially important for studies of thermal electron runaway phenomena (e.g. Chanrion and Neubert (2010), Colman et al (2010)). Since the RBEB formulation is analytical, it is directly implementable in Boltzmann or Monte Carlo codes, and since it does not require any adjustable parameters it is straightforward to apply it for different gases, for example, in order to simulate relativistic breakdown in planetary atmospheres (e.g. Dwyer (2007), Roussel-Duprê et al (2008)).

(ii) A direct comparison of the RREA rates obtained using the Møller and RBEB differential ionization cross sections have shown that the dipole interaction between primary electrons and K-shells of N and O leading to the generation of energetic secondary electrons has an impact on RREA rates for applied electric fields higher than $\approx 20$ kV cm$^{-1}$.

(iii) The Swann–Fermi density effect included in the database EEDL compensates the effect of soft ionizing collisions with K-shell electrons on RREA rates. Therefore, assuming that the density effect is correctly calculated in the EEDL implies that the RREAs developing at ground pressure are not directly scalable to those developing at
higher altitudes (i.e. cloud tops) where the density effect is negligible.

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