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Key Points:
- Air density is only one of the three factors affecting streamer dynamics
- Initial conditions and applied electric field are the other two factors
- Sprite streamers are smaller at lower altitudes due to branching

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Dynamics of sprite streamers in varying air density

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Abstract Similarity laws for streamer discharges, which state that the properties of streamers such as streamer radius, electric field, and electron density should respectively scale as N^{-1}, N^1, and N^2 in different but uniform air densities N, are important relations that have provided a general understanding of the mesospheric sprite discharges based on existing knowledge of streamers in laboratory conditions. Recent modeling studies, however, show that the properties of sprite streamers in varying air density do not follow exactly or even contradict the similarity relations. We present here simulation results and related analysis of sprite streamers to provide a unified view and resolve these contradictions. Our results indicate that the properties of streamers in varying air density are determined by the physical dimensions of streamers and the reduced electric field E/N, with the varying air density N being only one of the three factors controlling the streamer properties.

1. Introduction

Sprites are large-scale optical emissions induced by transient lightning electric fields in the Earth's upper atmosphere above active thunderstorms [*Sentman et al.*, 1995; *Lyons*, 1996]. In high-speed video observations, sprites usually consist of a large diffuse glow at the top, referred to as a sprite halo, and filamentary streamers at the lower tendril part of the sprites [*Stanley et al.*, 1999; *Stenbaek-Nielsen et al.*, 2000]. Sprite streamers are initiated at ~80 km altitude from preexisting plasma irregularities in the lower ionosphere [*Qin et al.*, 2011, 2014; *Fullekrug et al.*, 2013], and they could propagate upward to ~90 km and downward to ~40 km altitude, covering almost the entire altitude range of the mesosphere [*Stenbaek-Nielsen et al.*, 2013]. Streamers developing over such a large spatial scale experience significant (~3 orders of magnitude) air density variation and exhibit complex processes of development such as exponential growing, decaying, colliding, and splitting [*Cummer et al.*, 2006; *Stenbaek-Nielsen et al.*, 2013], providing an excellent opportunity for studies of the dynamics of streamers in varying air density.

The similarity laws for streamer breakdown are a set of relations proposed originally to explain the nature of the filamentary structures in sprites as being essentially the same as the streamers observed in laboratory conditions [*Pasko et al.*, 1998; *Ebert et al.*, 2010]. The similarity relations were further investigated by *Liu and Pasko* [2004] emphasizing the nonsimilarity introduced by the process of photoionization to the scaling properties of sprite streamers when compared to those at the ground level. Experimental investigation also demonstrated the validity of the similarity laws in correlating the properties of streamers in different but uniform neutral densities by measuring the minimal diameters of positive streamers in laboratory conditions [*Briels et al.*, 2008]. On the other hand, the modeling results published by *Luque and Ebert* [2010] on the dynamics of sprites in varying air density show that the radius of downward sprite streamers grows instead of decreasing and that electron density in the streamer heads increases approximately as N rather than N^2 expected from similarity relations at constant air density. More recently, a new streamer propagation mechanism has been proposed by *Qin and Pasko* [2014], showing that in a region with uniform air density the streamer dynamics are controlled not solely by the applied electric field but also by the physical dimensions of the streamers. How streamers propagate in varying air density is an important science question, not completely addressed yet in the scientific literature. In the present work, we use the streamer model developed by *Qin et al.* [2013] to provide a unified view on the properties of sprite streamers propagating in varying air density.

2. Model

A two-dimensional cylindrically symmetric (r, z dependent) plasma fluid model developed by *Qin et al.* [2013] is used to simulate the time dynamics of sprite streamers. In this model, the chemical reactions accounted for include electron impact ionization of N_2 and O_2, electron dissociative attachment to O_2, and the electron detachment process $O^- + N_2 \rightarrow e + N_2O$ [*Luque and Gordillo-Vazquez*, 2012; *Liu*, 2012; *Qin et al.*, 2013, equations (1)–(4)]. Photoionization processes are included using the three-group SP_3 model developed by *Bourdon et al.* [2007]. The motion of charged species is simulated by solving the drift-diffusion equations for electrons and ions coupled with Poisson's equation [e.g., *Qin et al.*, 2013, equations (5)–(9)]. The transport equations for charged species are solved using a flux-corrected transport technique that combines an eighth-order scheme for the high-order fluxes and a donor cell scheme for the low-order fluxes. Optical emissions are calculated using the model documented by *Liu and Pasko* [2004]. In all our simulations, the simulation domain extends from 75 km to 80 km altitude (a factor of 2.2 air density variation) in the vertical direction with a radius of 0.625 km and is discretized using a grid 2400×300 that corresponds to a uniform spatial resolution of 2.08 m. The vertical 5 km extent of the simulation domain is comparable with e-folding length ~7.2 km of the ambient air density variation.

3. Results and Discussion

3.1. Understanding of Streamer Properties in Varying Air Density

Previous modeling study of *Liu and Pasko* [2004] shows that for streamers propagating in different but uniform air densities, the properties of streamers scale following the similarity relations if their initiation conditions (e.g., applied electric field, size, and density of the initial plasma column) follow exactly the same relations. Those similarity relations have been summarized by *Pasko* [2006], some of which are listed below: in different but uniform air densities N, streamer length, radius and electron mobility μ_e scale as N^{-1}, ionization frequency ν_i and electric field E scale as N, and electron density n_e scales as N^2. *Luque and Ebert* [2010] assumed constant electric field of 40 V/m at sprite altitudes and performed simulations of sprite streamers in varying air density. They found that the radius of downward streamers grows instead of decreasing and that the electron density in the streamer head increases approximately as N rather than N^2. These results, as interpreted by *Luque and Ebert* [2010], seem to contradict the similarity laws.

To better understand the dynamics of streamers in varying air density, we conduct streamer simulations similar to those of *Luque and Ebert* [2010], except that we also modify the applied electric field to study its impact on the streamer dynamics in varying air density. We first present two cases that are respectively related to constant electric field of 40 V/m (i.e., the same as that assumed by *Luque and Ebert* [2010]) and constant reduced electric field $E/E_k = 0.8$, where E_k is the breakdown field that is proportional to N. A spherical plasma inhomogeneity is placed slightly below 80 km altitude to initiate sprite streamers, which has a Gaussian spatial distribution with a characteristic size of 80 m and peak density of 2×10^9 m^{-3}. Note that additional tests (not presented) show that using different initial inhomogeneities would not affect the conclusions made in this work. In the case of $E = 40$ V/m, the electron density in the downward streamer head increases approximately as N, in agreement with the results published by *Luque and Ebert* [2010]. In the case of $E = 0.8E_k$, the electron density in the streamer head increases approximately as $N^{2.4}$, as shown in Figure 1a. In both cases, we find that the radius of the downward streamer increases, as illustrated in Figures 1b–1d using the case of constant reduced electric field. These two cases illustrate that the applied electric field has a significant impact on the properties of streamers in varying air density.

We emphasize that the above results should not be interpreted as contradiction with the similarity laws. Increase of streamer radius at lower altitudes with higher air density, instead of decrease as one might expect from the similarity laws, can be understood based on the streamer propagation mechanism proposed by *Qin and Pasko* [2014]. The expectation that the size of streamers in higher air density should be smaller assumes that streamer properties are determined solely by the air density N. However, even in a uniform air density N, for different applied electric fields streamers could experience exponential growth, decay, or stable propagation, leading to significant variation of their radius, electron density, and other properties. According to *Qin and Pasko* [2014], in uniform air density the streamer dynamics (thus the streamer properties at each moment of time) are controlled by the external field and the physical dimensions of streamers. Allowing variation of the air density merely introduces one additional variable affecting the propagation of streamers, and thus, a more general statement of the streamer propagation

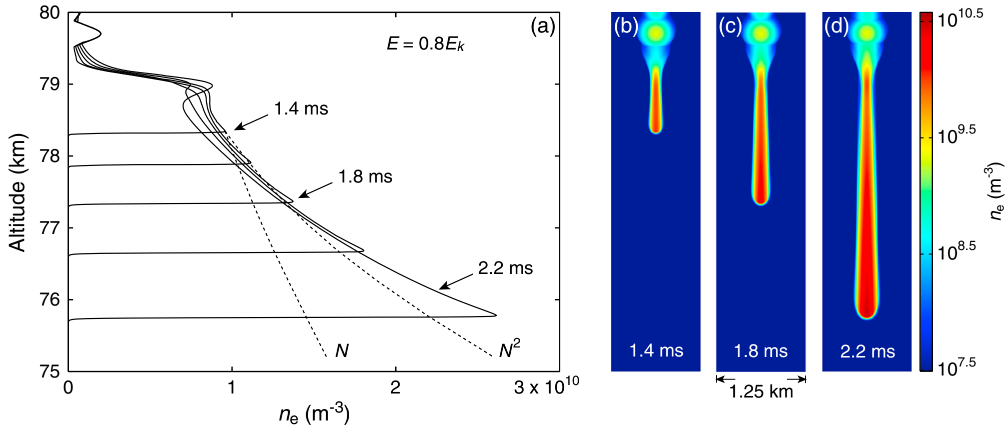

Figure 1. (a) Electron density on the axis of symmetry of the downward positive streamer produced by a constant reduced electric field $E/E_k = 0.8$. (b–d) Cross-sectional view of electron density of the same streamer at different moments of time.

mechanism should be as follows: the streamer dynamics are controlled by the physical dimensions of streamers, which are determined by the history of streamer development, and the reduced electric field E/N. In our simulations shown in Figure 1, the applied electric field is higher than the stability field, which is the requirement for growing streamers [*Qin and Pasko*, 2014]. Based on the above discussion, the dimension of streamer head in varying air density is not inversely proportional to the air density N but is significantly affected by the history of streamer development at earlier moments of time and the reduced electric field E/N. It should be emphasized that although higher air density does not lead to smaller streamers at lower altitudes directly through forcing the streamers to propagate in a decaying manner, higher air density could lead to easier streamer branching due to shorter photoionization range [*Liu and Pasko*, 2004] and thus lead to smaller streamers at lower altitudes.

To explain why the electron density in the streamer head does not scale as N^2 (see Figure 1), we first need to understand the physical parameters that determine the electron density in streamers. During the propagation of streamers, free electrons are produced in the high-field region of the streamer head mainly through ionization. The increase of electron density in this high-field region stops when the Maxwell relaxation timescale τ is comparable to the ionization time ν_i^{-1}:

$$\frac{\varepsilon_0}{q_e \mu_e n_e} \simeq \frac{1}{\nu_i} \tag{1}$$

where ε_0 is the permittivity of free space and μ_e is the electron mobility, and thus,

$$n_e \sim \frac{\nu_i}{\mu_e} \tag{2}$$

in which ν_i scales as N, μ_e scales as N^{-1}, and thus n_e scales as N^2. Since both ν_i and μ_e are functions of the reduced electric field E/N, the scaling relations are valid only if the reduced electric field remains constant. Nonconstant reduced electric field leads to deviation of the electron density from scaling as N^2 when propagating in varying air density. To substantiate the above discussion, we show in Figure 2 the variation of the electron density and reduced electric field in the streamer head in three different simulation cases. In the case of constant applied electric field $E = 40$ V/m, the electron density in the downward streamer head increases as $\sim N$ (slower than N^2; see Figure 2a), which results from a decreasing reduced electric field in the streamer head (see Figure 2b). On the contrary, in the case of constant reduced electric field $E = 0.8E_k$, the reduced electric field in the streamer head increases, thus leading to the increase of electron density in the streamer head as $N^{2.4}$ (faster than N^2). It is therefore expected that if the reduced electric field in the streamer head remains constant, the electron density should vary exactly as N^2. A parametric study has been performed to find the required applied electric field, which for simplicity is assumed to vary linearly with altitude:

$$E(h) = 42.4 \times \frac{h - 79}{7.2} - 40 \quad \text{V/m} \tag{3}$$

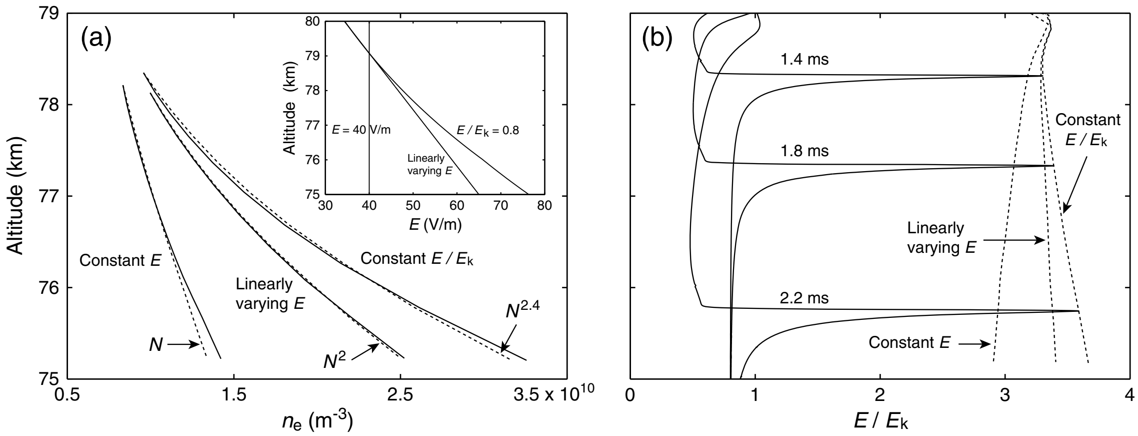

Figure 2. (a) Variations of the peak electron density in the head of modeled downward propagating sprite streamers. Solid curves from left to right: the case of constant electric field $E = 40$ V/m, linearly varying electric field (see equation (3)), and constant reduced electric field $E/E_k = 0.8$. Dashed lines show the fitting of the curves using the variation of air density N. The inserted figure shows the applied electric fields. (b) Solid lines show the electric fields at different moments of time on the axis of symmetry of the sprite streamer produced in the case of constant $E/E_k = 0.8$. Dashed lines show the variation of the peak electric field in the streamer head in the three different cases.

where h (km) is the altitude of interest and 7.2 km is the e-folding length of the ambient air density variation. From the results shown in Figure 2, we see that in this case the reduced electric field in the streamer head remains constant and the electron density varies as N^2, in agreement with the scaling relations.

3.2. Properties of Sprite Streamers in Lightning Electric Fields

Spatial variation of lightning-induced electric field at sprite altitudes as h^{-3} is more complicated than the cases assumed in section 3.1. The reduced lightning electric field thus has the form of $h^{-3}e^{h/h_0}$, where $h_0 \simeq 7.2$ km is the e-folding length of the air density variation. Moreover, the lightning electric field also varies over time during the discharge process due to increase of the charge removed from the thundercloud and dielectric relaxation at sprite altitudes. To more accurately model the properties of sprite streamers, temporal and spatial variations of the lightning electric field need to be taken into account. This is fulfilled by using the two-step technique proposed by *Qin et al.* [2012], in which a large-scale halo model is implemented first to acquire the temporal and spatial variations of the lightning electric field, and then the lightning electric field is applied to initiate sprite streamers from plasma inhomogeneities.

Figure 3 shows the modeled emissions of the first positive band system of $N_2(1PN_2)$ for two positive streamers produced by positive cloud-to-ground lightning discharges that are, respectively, associated with charge moment changes of 400 C km and 600 C km. The plasma inhomogeneities used to initiate streamers

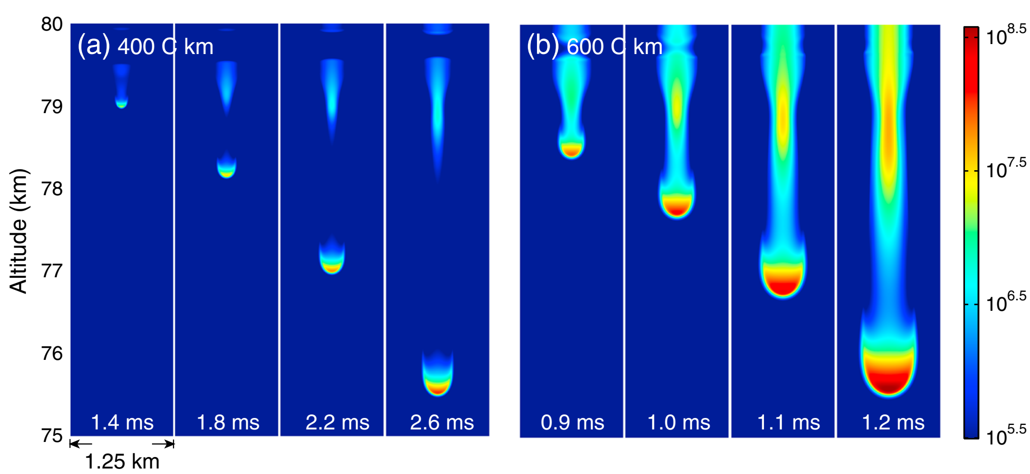

Figure 3. Cross-sectional views of optical emissions (Rayleighs) in the positive streamers, respectively, produced by lightning discharges of 400 C km and 600 C km.

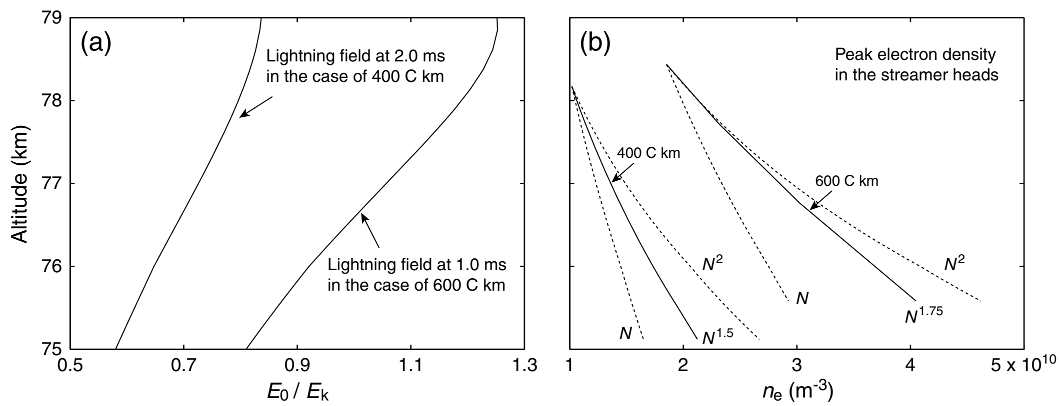

Figure 4. (a) Spatial variation of the electric fields induced by the two model lightning discharges, at the moments after the streamers have been initiated. (b) Variations of the peak electron density in the heads of modeled downward propagating sprite streamers shown in Figure 3.

have a Gaussian distribution with a characteristic size of 150 m and a peak density of 2×10^9 m^{-3}. In both cases the radius of streamers increases with decreasing altitude. In the case of 600 C km, the streamer radius increases more rapidly due to higher lightning electric field, and at 1.2 ms the streamer radius is ∼300 m which almost reaches the median value for splitting streamers in the observations of *McHarg et al.* [2010]. We have also analyzed how the streamer velocity varies with time. In the case of 400 C km, the velocity of the downward streamer increases almost linearly over time from 2.04×10^6 m/s at 1.6 ms to 4.17×10^6 m/s at 2.7 ms, with an acceleration rate of $\sim 2 \times 10^9$ m/s^2, whereas in the case of 600 C km, the streamer velocity increases from 1.19×10^6 m/s at 0.7 ms to 1.19×10^7 m/s at 1.2 ms, corresponding to an acceleration rate of $\sim 2 \times 10^{10}$ m/s^2. The increase of streamer radius and velocity when propagating to higher air density region, as explained in section 3.1, shows that these two streamer parameters are controlled by the history of the streamer development, which determines the physical dimensions of streamers, and the reduced electric field E/N, but not solely dictated by the ambient air density N.

Figure 4a shows the lightning electric fields at selected moments of time in the two modeled cases, and Figure 4b shows the variation of the peak electron density in the streamer head with altitude. We know that the lightning electric field increases with decreasing altitude as h^{-3}. However, this increase is slower than the exponential increase of air density N with decreasing altitude, which leads to a decreasing reduced electric field E_0/E_k as shown in Figure 4a. The decreasing reduced lightning electric field E_0/E_k leads to a decreasing reduced electric field in the streamer heads, and thus, the peak electron density in the streamer heads respectively varies as $N^{1.5}$ and $N^{1.75}$, slower than as N^2 that requires a constant reduced field in the streamer head.

4. Conclusions

In this study, we investigated the properties of streamers in varying air density at sprite altitudes to show how the similarity laws for streamer discharges, originally formulated in the case of different but uniform air densities, could be understood in the case of varying air density. We explain that the properties of streamers in different but uniform air densities could scale with each other following exactly the similarity relations only if their initiation conditions also scale with each other following the same relations. For streamers propagating in varying air density, scaling of the initial conditions in different air densities cannot be fulfilled and thus the streamer properties could not follow the scaling relations. The streamer properties at a given moment of time are significantly affected by the history of the streamer development and the reduced electric field E/N, with the varying air density being only one of the three factors controlling the streamer dynamics (other two factors are the applied electric field and the physical dimensions of streamers). We have also modeled sprite streamers produced by lightning electric fields. The results show that the radius of sprite streamers increases when propagating into lower altitudes with higher air density, which suggests that streamers become smaller at lower altitudes mainly due to streamer branching instead of decaying.

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