

Minimum charge moment change in positive and negative cloud to ground lightning discharges producing sprites

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[1] Measurements indicate that surprisingly small charge moment changes of ~ 200 C km in positive cloud-to-ground lightning discharges (+CGs) can initiate middle atmospheric gas discharges termed sprites. In the present work a plasma fluid model is used to demonstrate that for spherically symmetric initial electron density inhomogeneities, the initiation of sprites by such small charge moment changes is only possible when the ionospheric D-region electron density profile is characterized by a reference altitude h' greater than 90 km. Vertically elongated inhomogeneities are found to be more favorable for sprite initiation, which is consistent with recently published studies. It is calculated that for the same ionospheric conditions (i.e., inhomogeneities and h' values) that lead to initiation of sprites by +CGs associated with ~ 200 C km charge moment changes, the minimum charge moment change required for initiation of sprites by -CGs is 300 C km. **Citation:** Qin, J., S. Celestin, and V. P. Pasko (2012), Minimum charge moment change in positive and negative cloud to ground lightning discharges producing sprites, *Geophys. Res. Lett.*, 39, L22801, doi:10.1029/2012GL053951.

1. Introduction

[2] Sprites [Sentman *et al.*, 1995] with extensive vertical streamer structures [e.g., Stanley *et al.*, 1999] are usually produced by the most intense (i.e., large peak current) positive cloud-to-ground lightning discharges (+CGs) in thunderstorms [Lyons, 1996]. In the theory proposed by Pasko *et al.* [1997], the quasi-static electric field that generates sprites is linearly proportional to the charge moment change Qh_Q of the cloud-to-ground lightning discharge, which is defined as the amount of charge Q transferred to ground by the lightning discharge times the altitude h_Q from which the charge was removed. In this theory a large charge moment change of ~ 1000 C km, for which the electric field can exceed the conventional breakdown field E_k (~ 30 kV/cm at ground level) at mesospheric/lower ionospheric altitudes, is usually assumed to be necessary for sprite production [e.g., Pasko *et al.*, 1997].

[3] Lightning charge moment change at the time of sprite initiation was measured at Duke University during the Summer of 2000 [Hu *et al.*, 2002; Cummer and Lyons, 2005]. It was found that the majority of sprites were produced

by +CGs associated with $Qh_Q \gtrsim 600$ C km up to thousands of C km, in general agreement with the quasi-static electric theory [Pasko *et al.*, 1997]. However, Hu *et al.* [2002] also indicated that +CGs associated with $Qh_Q < 600$ C km still have $< 10\%$ probabilities to produce sprites, and most surprisingly that sprites can be initiated by +CGs associated with Qh_Q as small as 120 C km.

[4] Successful numerical modeling of sprite streamer inception in the case of empirical minimum charge moment changes has not been achieved so far, and it represents a major goal of the present work. Since the charge moment change, ambient conductivity profile, and electron density inhomogeneities in the lower ionosphere are all critical for the streamer initiation at sprite altitudes [Qin *et al.*, 2011], it is not possible to calculate theoretically the minimum charge moment change required to produce sprites due to the poorly known lower ionospheric variations. Therefore, in the case of 'positive sprites' (produced by +CGs), a value of 200 C km is selected based on the observations of Hu *et al.* [2002] as a reference minimum charge moment change in order to find the corresponding requirements of lower ionospheric conditions (i.e., ambient electron and ion density profiles, inhomogeneities) for enabling streamer initiation. We further apply these ambient conditions to the case of -CGs in order to predict the minimum charge moment change required to produce 'negative sprites.' Note that we choose the threshold value of 200 C km rather than 120 C km because the sprite event with $Qh_Q \approx 120$ C km reported by Hu *et al.* [2002] appears to be an exceptional case, whereas these with $Qh_Q \approx 200$ C km are repeatedly observed in North America [e.g., Hu *et al.*, 2002], as well as at other locations, such as in the Hokuriku area of Japan [e.g., Hayakawa *et al.*, 2004].

2. Model Formulation

[5] A two-dimensional cylindrically symmetric (r, z dependent) plasma fluid model developed by Qin *et al.*, 2012b, also Dependence of positive and negative sprite morphology on lightning characteristics and upper atmospheric ambient conditions, submitted to *Journal of Geophysical Research*, 2012] is used to simulate the time dynamics of sprite halos and sprite streamers. In this model, the chemical reactions accounted for include electron impact ionization of N_2 and O_2 , electron dissociative attachment to O_2 , and the electron detachment process $O^- + N_2 \rightarrow e + N_2O$ [Qin *et al.*, 2012b, equations (1)–(4)]. Photoionization processes are included using the three-group SP₃ model developed by Bourdon *et al.* [2007]. The motion of charged species is simulated by solving the drift-diffusion equations for electrons and ions coupled with Poisson's equation [Qin *et al.*, 2012b, equations (5)–(9)]. The transport equations for charged species are solved using a flux-corrected transport technique [Zalesak,

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1979] that combines an eighth-order scheme for the high-order fluxes and a donor cell scheme for the low-order fluxes. The ambient ionospheric electron density profile is expressed as follows [Wait and Spies, 1964]:

$$n_e(h) = 1.43 \times 10^{13} e^{-0.15h'} e^{(\beta-0.15)(h-h')} \quad (1)$$

where h' and β are given parameters describing reference altitude and sharpness, respectively. In the present work, we fix $\beta = 0.5 \text{ km}^{-1}$ that was shown to be typical during nighttime [Han and Cummer, 2010] and investigate the lowest h' that is required for sprite initiation with $Qh_Q = 200 \text{ C km}$. Note that the ambient electron density increases exponentially with altitude. When plotting equation (1) on a log-lin scale, it is precisely a straight line. Modification of h' leads to increase or decrease of the ambient electron density by the same magnitude in the whole domain [see, e.g., Han and Cummer, 2010, Figure 2 (top)]. Typical nighttime values of h' range from 82.0 to 87.2 km [Han and Cummer, 2010]. The ambient positive ion density equals the electron density at high altitudes where electron density is higher than 10^8 m^{-3} , and is 10^8 m^{-3} at low altitudes [e.g., Qin et al., 2012b]. The ambient negative ion density is then calculated based on charge neutrality. The drift of ions is incorporated assuming the mobility of ions as a function of altitude $\mu_i \approx 2.3N_0/N \text{ cm}^2/\text{V/s}$ [Davies, 1983], where N is the air density at altitude of interest and $N_0 \approx 2.688 \times 10^{25} \text{ m}^{-3}$ is its reference value at ground level.

[6] A ‘two-step’ simulation technique proposed by Qin et al. [2012a] is used in order to model the initiation of streamers from sprite-halo events. In the first step, we model the large-scale sprite halo dynamics in a simulation domain that extends from the ground up to 95 km, with a radius of 95 km and perfectly conducting boundary conditions on the upper (ionosphere), lower (ground), and lateral (95 km away from the center) boundaries using a numerical grid with a spatial resolution of 237.5 m. We keep track of the electric field $\vec{E}_{\text{halo}}(r, z, t)$ in the upper atmosphere during this step, and then use this field as an externally applied electric field during the second step to model possible small-scale sprite streamer initiation using a much finer numerical grid. In the case of +CGs/−CGs, the simulation domain in the second step extends 2 km/4 km vertically and has a radius of 0.25 km/0.50 km with open boundary conditions, and it is discretized in different simulations using grids with $3201 \times 401/6001 \times 751$ grid points, corresponding to a spatial resolution of 0.625 m/0.667 m. The variation of air density with altitude is accounted for, and the initial electron and ion densities in the streamer simulation domain are the same as those in the corresponding region in sprite halo modeling, defined by the equation (1). To monitor the possible streamer initiation, we place a test inhomogeneity at different altitudes on the axis of the halo, and use a streamer model to simulate its evolution under application of $\vec{E}_{\text{halo}}(r, z, t)$. The test inhomogeneities have Gaussian density distributions:

$$n_{\text{inhomo}} = n_{\text{peak}} \exp \left[-\frac{r^2}{r_0^2} - \frac{z^2}{z_0^2} \right] \quad (2)$$

where n_{peak} , r_0 and z_0 are the peak density, and the characteristic size of the inhomogeneity in radial and axial directions, respectively.

[7] The lightning current moment waveform is modeled using the formulation proposed by Cho and Rycroft [1998]:

$$Ih_Q(t) = \frac{Qh_Q}{12t_0} \left(\frac{t}{t_0} \right) \exp \left[-\left(\frac{t}{t_0} \right)^{1/2} \right] \quad (3)$$

where $t_0 = 25 \mu\text{s}$ is used in this study.

3. Results and Discussion

3.1. Spherical Inhomogeneities Leading to Positive Sprites

[8] We first conduct a parametric study to find out the upper atmospheric ambient conditions, namely the strength of preexisting electron inhomogeneities in the lower ionosphere and the D-region electron density profile, required for a +CG associated with a minimum charge moment change of 200 C km to produce sprites. For $Qh_Q = 200 \text{ C km}$, it is expected that the streamer initiation region (SIR) will be located above $\sim 80 \text{ km}$ and up to $\sim 85 \text{ km}$ altitude, since this is the region with large lightning-induced electric field $E \geq 0.5E_k$ according to our sprite halo modeling (i.e., the first step of our two-step simulations) [Qin et al., 2012a]. In the sprite streamer modeling (i.e., the second step), in order to search for a reasonable ambient electron density profile that leads to streamer initiation by such a small charge moment change, we assume $n_{\text{peak}} = 2 \times 10^9 \text{ m}^{-3}$ to be the peak density of the initial inhomogeneity (see equation (2)), which is almost as high as the electron density in a streamer body at $\sim 84 \text{ km}$ altitude, and r_0 and z_0 to be tens of meters, which are close to the initial size of streamers at these altitudes according to the similarity laws [Pasko et al., 1998].

[9] A spherically distributed inhomogeneity that is characterized by $n_{\text{peak}} = 2 \times 10^9 \text{ m}^{-3}$ and $r_0 = z_0 = 30 \text{ m}$ in equation (2) is assumed in the first case study. With this inhomogeneity and a fixed $\beta = 0.5 \text{ km}^{-1}$ in equation (1), we found that a value of $h'=91 \text{ km}$ is the lowest reference altitude in the electron density profile that allows streamer initiation. Figure 1a shows the simulated optical emissions of the first positive band system of N_2 (1PN₂) as well as the reduced electric field E/E_k on the axis of symmetry for a streamer initiated from this inhomogeneity under the conditions defined by $Qh_Q = 200 \text{ C km}$ and $h'=91 \text{ km}$. We note that this streamer develops more slowly in its early stage when compared to those simulated by Liu et al. [2009] and Qin et al. [2012a], and it propagates only $\sim 200 \text{ m}$ during a 2.0 ms period from $t = 3.0 \text{ ms}$ to 5.0 ms . This effect can be directly attributed to lengthening of characteristic time scales in streamer with air density $\sim 1/N$ (i.e., the streamer shown in Figure 1a is initiated at lower air density at 84.25 km altitude, whereas these modeled by Liu et al. [2009] and Qin et al. [2012a] were initiated at $\sim 75 \text{ km}$ altitude). Qin et al. [2012b] have demonstrated that the long persistence of the lightning-induced electric field is critical for sprite streamer initiation, and to some extent, can be considered even more important than the absolute value of the electric field. The low air density above 85 km that leads to slow streamer development, is an important factor that impedes streamer initiation, since long persistence of the lightning-induced electric field

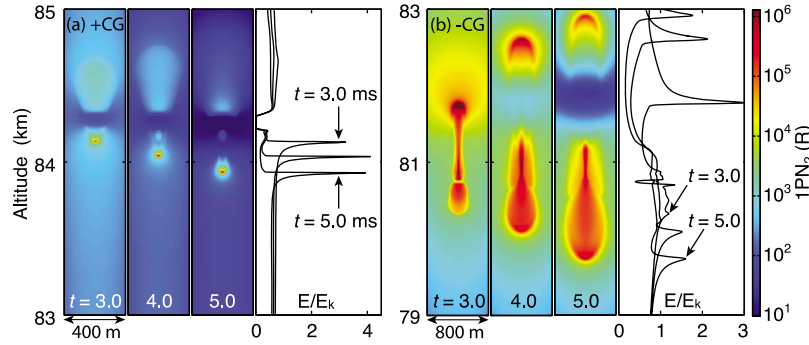


Figure 1. Cross-sectional view of optical emissions and reduced electric field E/E_k on the axis of symmetry for a streamer produced by (a) a +CG associated with Qh_Q of 200 C km, and (b) a -CG associated with Qh_Q of 300 C km. All the other conditions in these two cases are the same. The electron density profile is characterized by $h' = 91$ km and $\beta = 0.5$ km $^{-1}$ in equation (1), and the initial inhomogeneity is characterized by $n_{\text{peak}} = 2 \times 10^9$ m $^{-3}$ and $r_0 = z_0 = 30$ m in equation (2).

is even less likely at these high altitudes (i.e., cases for which $h' \gtrsim 91$ km). We note that the sprite onset altitude of ~ 84 km is consistent with triangulation measurements of *Stenbaek-Nielsen et al.* [2010] and *Kanmae et al.* [2012] showing that sprites could have onset altitudes as high as 88 km.

[10] Moreover, we have confirmed that increasing the peak density of the inhomogeneity (e.g., up to $n_{\text{peak}} = 2 \times 10^{10}$ m $^{-3}$) would not lower the requirement on high h' for streamer initiation because the peak density of 2×10^9 m $^{-3}$ is already as high as the electron density in a streamer channel at ~ 84 km. A larger radius of the inhomogeneity, such as $r_0 = z_0 = 100$ or 200 m, only slightly lowers the minimum h' down to 90 km. This is because a value of 100 m matches closer to the initial radius of streamers at ~ 84 km altitude. We conclude that in the case of the most favorable spherical inhomogeneities in the lower ionosphere in this model, $h' = 90$ km is the lowest reference altitude in the profile of ionospheric electron density that allows sprite streamer initiation by a +CG associated with a charge moment change of 200 C km.

3.2. Elliptical Inhomogeneities Leading to Positive Sprites

[11] *Kosar et al.* [2012] have indicated that the shape of the inhomogeneity affects sprite streamer initiation based on their modeling results. Therefore, in this work, we have introduced a vertically elongated initial inhomogeneity assuming that $n_{\text{peak}} = 2 \times 10^9$ m $^{-3}$, $r_0 = 30$ m and $z_0 = 90$ m ($z_0/r_0 = 3$). In this case, we find that the lowest reference altitude that allows streamer initiation with $Qh_Q = 200$ C km is $h' = 88$ km. This lower minimum reference altitude, when compared to $h' = 90$ km in the case of spherical inhomogeneities, indicates that elliptical inhomogeneities with $z_0 > r_0$ are more favorable for sprite streamer initiation. This is due to the fact that the initial electric field created by polarization of the electron inhomogeneity is more intense in the case of elliptical inhomogeneities oriented in the direction of the ambient field [*Kosar et al.*, 2012]. We have tested that when simultaneously increasing r_0 and z_0 with a constant ratio $z_0/r_0 = 3$, for example $r_0 = 100$ m and $z_0 = 300$ m, streamer initiation would still require the same low ambient electron density (i.e., $h' = 88$ km). However, increasing the ratio of z_0 and r_0 , for example with $r_0 = 30$ m and $z_0 = 120$ m, would indeed enable streamer initiation with $h' = 87$ km, consistent with conclusions of *Kosar et al.* [2012] that the threshold field E for streamer initiation is lower for vertically elongated inhomogeneities.

3.3. Minimum Charge Moment Change Producing Negative Sprites

[12] The above-discussed upper atmospheric ambient conditions (i.e., $h' = 91$ km and $z_0 = r_0 = 30$ m, or $h' = 88$ km and $z_0 = 3r_0 = 90$ m, either of which leads to production of positive sprites by the minimum Qh_Q of 200 C km) are the most favorable conditions that should exist in the lower ionosphere for sprite production. It is well known that +CGs associated with only $Qh_Q = 200$ C km produce positive sprites very infrequently [e.g., *Hu et al.*, 2002; *Cummer and Lyons*, 2005]. We emphasize that these two different sets of conditions lead to the same minimum Qh_Q of 200 C km producing positive sprites. This indicates that the unfavorable condition introduced by lowering h' from 90 km to 88 km is compensated by increasing the ratio of z_0/r_0 from 1 to 3 in the case of +CGs.

[13] Using these most favorable conditions in our two-step simulations, we find that a minimum Qh_Q of 300 C km is required to initiate ‘negative’ sprites in the case of $h' = 91$ km and $z_0 = r_0 = 30$ m, or 360 C km is required if the ambient conditions defined by $h' = 88$ km and $z_0 = 3r_0 = 90$ m in the lower ionosphere occur. Interestingly, these two sets of conditions lead to different minimum Qh_Q producing ‘negative’ sprites, both of which are larger than that 200 C km required to produce positive sprites. Figure 1b shows the dynamics of a streamer initiated by a -CG associated with $Qh_Q = 300$ C km in the case of $h' = 91$ km and $z_0 = r_0 = 30$ m. It appears that the downward negative streamer shown in Figure 1b is much more diffuse than the upward positive streamer shown in Figure 1a, as well as the downward positive streamer shown in Figure 1a. This is because the negative space charge, which defines the location and shape of the negative streamer head, is formed by radially spreading electrons, whereas the positive space charge in the positive streamer head is formed by almost motionless ions. This, combined with different directions of electron drift with respect to applied electric field leads to more compact size and higher electric field in positive streamer head in comparison with the negative one [e.g., *Liu and Pasko*, 2004] as evident in Figure 1. During the inception stage, the space charge in the negative streamer head is distributed in a larger volume and thus creates smaller electric field, when compared to that in the positive streamer head. Since the space charge field in the negative streamer head is not as strong as that in the positive streamer head, increasing the ratio z_0/r_0

from 1 to 3 does not compensate the unfavorable conditions introduced by lowering h' from 90 km to 88 km. Moreover, the weaker space charge field in negative streamers is also responsible for the fact that the initiation of 'negative' sprites requires larger charge moment change (i.e., higher external field) than that for positive sprites, since the total electric field in streamer heads must be higher than the conventional breakdown field E_k to create a region dominated by ionization.

3.4. Rarity of Negative Sprites and the Importance of Lower Ionospheric Conditions

[14] It appears that the occurrences of -CGs associated with $Qh_Q \gtrsim 300$ C km are not rare according to *Williams et al.* [2007]. More recently, *Lu et al.* [2012] analyzed 2126 -CGs with peak currents over -80 kA in a 100×100 km² area centered at the origin of the north Alabama lightning mapping array, and documented that 13 (0.6%) events produced charge moment changes larger than 300 C km and up to 420 C km. On the other hand, only nine negative sprite events have been documented in the existing literature [*Barrington-Leigh et al.*, 1999; *Taylor et al.*, 2008; *Li et al.*, 2012]. Using the results obtained in the present study, this suggests that h' may rarely get higher than ~ 88 km, which is a requirement in order to produce negative sprites in the case of $Qh_Q \approx 300$ C km. Indeed, *Han and Cummer* [2010] probed the ionospheric D-region by measuring the high-power broadband VLF signals generated by lightning and propagating in the Earth-ionosphere waveguide in July and August of 2005, and found that the measured hourly average nighttime D-region electron density profile reference altitude h' in 260 hours ranged between $h' = 82.0$ and 87.2 km, with a mean value of 84.9 km. We suggest that the intersection of the rarity of $Qh_Q \approx 300$ C km and $h' \gtrsim 88$ km leads to the fact that the occurrence of negative sprites are rare, whereas -CGs associated with $Qh_Q \approx 300$ C km are not so rare.

[15] We note that with a typical nighttime value of h' , such as $h' = 85$ km [*Han and Cummer*, 2010], production of negative sprites are still possible. However, as calculated by *Qin et al.* [2012b], it requires large charge moment changes of at least 500 C km, which are rare according to the analysis of *Williams et al.* [2007, 2012, Figure 8], and does not even occur in the measurements reported by *Lu et al.* [2012]. Note that we have calculated that it requires a minimum of 320 C km for +CGs to initiate sprites under typical nighttime conditions [*Qin et al.*, 2012b]. It should also be emphasized that in the discussion of *Qin et al.* [2012b] and in the present study it is assumed that a strong inhomogeneity is present in the ambient ionosphere, that by itself may represent a significant limiting factor for initiation of sprites. This also directly reiterates the idea that the charge moment change alone is a necessary but not sufficient factor for sprites [*Lang et al.*, 2011] and the state of the lower ionosphere and presence of inhomogeneities are important factors, especially for low charge moment changes.

[16] Lastly, we note that the interpretation of *Qin et al.* [2012b] and the present study for the polarity asymmetry of +CGs and -CGs in producing sprite streamers based on the streamer physics and the interpretation of *Williams et al.* [2012] based on the different lightning current timescales in +CGs and -CGs are fundamentally different. *Williams et al.* [2012] found that -CGs primarily produced halos with numbers sufficient to account for the previously missing transient luminous events, and speculated that the more

impulsive characteristic of the lightning current in these halo-related -CGs is the reason for the rarity of negative sprites, while *Qin et al.* [2011] have concluded that the total charge moment change is the key parameter that determines the ability of a lightning discharge in producing sprites [see *Qin et al.*, 2011, paragraph [45]] and *Qin et al.* [2012b] have indicated that the more impulsive characteristic of -CGs is favorable for streamer initiation since it enhances the mesospheric electric field, and the difference between the threshold charge moment changes required for the initiation of the positive and negative sprites is the answer to the sprite paradox presented by *Williams et al.* [2007]. We also note that in a recently published work, *Li et al.* [2012] have observed that very impulsive -CGs (producing at least 450 C km charge moment change in 0.5 ms or less) associated with essentially no continuing current could indeed produce negative sprites. The charge moment changes responsible for the 6 negative sprites were 750, 1050, 600, 650, 460, 450 C km, in remarkable agreement with a prediction of ~ 500 C km threshold in *Qin et al.* [2012b]. We emphasize that the difference between the charge moment thresholds required for the initiation of positive and negative sprites is only one of the three major factors accounting for the polarity asymmetry of sprites [*Qin et al.*, 2012b]. Observability of sprites is another critical factor as streamers in sprites go through a significant growth before they become observable [*Liu et al.*, 2009; *Qin et al.*, 2012a]. The downward streamers in negative sprites appear to be dimmer than their upward counterparts due to a higher critical field required for the propagation of negative streamers [*Qin et al.*, 2011; *Li et al.*, 2012]. Many negative sprites may have been produced but left undetected by observation instruments. A simple estimate based on the charge moment change data documented by *Williams et al.* [2007] and the charge moment thresholds for positive and negative sprites derived by *Qin et al.* [2012b] (respectively 320 C km and 500 C km under typical nighttime conditions) leads to the theoretical result that $\sim 5\%$ of sprites should be negative. However, it should be noted that due to the differences in fields required for propagation/growth of streamers of different polarity and the resultant intrinsic dimness of negative streamers propagating in a given field as compared to positive streamers, this estimate must be considered as an upper limit for observations. More measurements and observations on the impulsiveness of sprite-producing -CGs, dim streamers in negative halos produced by -CGs, and the charge moment contrast of +CGs and -CGs are necessary to further test the theories of sprite asymmetry.

4. Conclusions

[17] Production of positive sprites by a +CG associated with a minimum charge moment change of 200 C km requires stringent upper atmospheric ambient conditions. If preexisting electron inhomogeneities in the lower ionosphere are spherically Gaussian distributed, the reference altitude h' of the D-region electron density profile needs to be higher than 90 km in order to enable positive streamer initiation. If elliptically Gaussian distributed inhomogeneities are present, a larger ratio of their vertical dimensions to radial dimension lowers the requirement of high h' . Using these stringent upper atmospheric ambient conditions, we calculate that 300 C km is the minimum charge moment change required for -CGs to

produce negative sprites. The difference between the charge moment thresholds required for the initiation of positive and negative sprites represents an important factor accounting for the polarity asymmetry of sprites.

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