A two-dimensional model of runaway electron beams driven by quasi-electrostatic thundercloud fields

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Abstract. Intense, transient quasi-electrostatic (QE) fields, which exist above thunderclouds following a positive cloud-to-ground lightning discharge, can produce an upward travelling runaway electron (REL) beam. A new two-dimensional (2D) REL-QE model is developed, expanding the previously reported 1D model [*Bell et al.*, 1995] and incorporating the QE [*Pasko et al.*, 1997a] and the recently developed electrostatic heating (ESH) [*Pasko et al.*, 1997b] models. The new model gives the lateral electron distribution in the beam and allows us to determine the ionospheric effects and the optical luminosities resulting from the simultaneous action of the QE fields on the ambient electrons and the runaway electrons. The model is self-consistent and includes the changes in space charge and conductivity due to the REL. Optical emissions and γ-ray emissions [*Lehtinen et al.*, 1996a] are calculated and compared to experimental observations of Sprites and terrestrial γ-ray flashes (TGF). It is shown that the structure of the electric field and the optical emissions can be significantly affected by the REL.

Introduction

Sprites are transient (lasting several ms) luminous glows occurring at altitudes $\sim$ 50 to 90 km and associated with energetic positive cloud-to-ground (+CG) discharges in underlying thunderstorms [*Lyons*, 1996; references therein]. Recent measurements of ELF/VLF radio atmospherics at Palmer Station, Antarctica [*Inan et al.*, 1996b] provide evidence of active thunderstorms near the inferred source regions of two different terrestrial γ-ray flashes (TGF) observed by BATSE detector on CGRO satellite [*Fishman et al.*, 1994], strongly indicating the presence of energetic electrons at mesospheric altitudes above thunderstorms [e.g., *Bell et al.*, 1995; *Lehtinen et al.*, 1996a].

Pasko et al. [1997a, hereafter referred as I; and references therein] proposed that sprites are produced by the heating of mesospheric electrons by large quasi-electrostatic (QE) thundercloud fields. The magnitude of these fields, predicted by the QE model [I], appears intense enough to produce a beam of $\sim$ 1 MeV runaway electrons (REL) [*Bell et al.*, 1995] through an avalanche mechanism [*Gurevich et al.* 1992; *Roussel-Dupré et al.*, 1994 and references therein]. In this work, we combine the previously reported 2D QE [I] and electrostatic heating (ESH) [*Pasko et al.*, 1997b] models with a new 2D REL model, which allows us to self-consistently describe the motion of the high energy (20 keV–10 MeV) REL in the electric $\vec{E}$ and geomagnetic $\vec{B}_0$ fields, their contribution to the QE field, and their effect on the atmospheric conductivity. We calculate the optical output and γ-ray emissions associated with the REL beam, and discuss model results in connection with Sprites and TGF. Analogous calculations were performed by *Roussel-Dupré and Gurevich* [1996] and *Taranenko and Roussel-Dupré* [1996]. The differences between their model and the present model are discussed below.

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Model

Our model describes processes which follow a slow (ms timescale) +CG lightning discharge in a conducting atmosphere. We utilize cylindrical coordinates (r, ϕ, z) with a vertical z-axis (axis of symmetry) and $\vec{B}_0 \parallel \hat{z}$. It should be noted that if $\vec{B}_0$ is tilted with respect to the vertical, then the REL beam is also tilted, following the magnetic field lines at high altitude.

Electric Field

We calculate the pre-discharge electrostatic field using the ESH model [*Pasko et al.*, 1997b]. The thundercloud charges $+Q$ and $-Q$ are located at heights 10 km and 5 km, respectively, and have Gaussian spatial distribution with a scale of $\sim$ 3 km each. The ambient conductivity of ions has an exponential profile of 6 km scale [*Dejnakarintra and Park*, 1974]. The slow +CG discharge is modelled by removing $+Q$ in a time of $\tau_s = 1$ ms, as described in [I]. The modified QE field equations, which account for the presence of REL, are [I]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_R = \frac{\rho_s \sigma_0}{\epsilon_0} \tag{1}$$

$$\nabla \cdot \vec{E} = \frac{\rho + \rho_s}{\epsilon_0}. \tag{2}$$

Here $\rho_s = \rho_s(t)$ is the source thundercloud charge density; ρ is the induced charge density, $\vec{J} = \hat{\sigma}\vec{E}$ is the conductivity current (conductivity tensor $\hat{\sigma}$ is dependent on E due to the heating process), $\vec{J}_R = -e\vec{v}_R N_R$ is the current due to REL with mean velocity $\vec{v}_R$ and number density N_R. The term on the RHS of the continuity equation (1) describes the external current $\nabla \cdot \vec{J}_{\text{ext}} = -\rho_s \sigma_0 / \epsilon_0$, supported by meteorological processes, with σ_0 being the specific conductivity [I].

Runaway Electrons

The REL transport equation can be written as:

$$\frac{\partial N_R}{\partial t} + \nabla \cdot (\vec{v}_R N_R) = \frac{N_R}{\tau_i} + S_o(z) \tag{3}$$

where $|\vec{v}_R| \sim c$ is the REL mean velocity, $1/\tau_i$ is the REL production rate, and S_o is the local source function for energetic cosmic ray secondary electrons [*Bell et al.*, 1995]. $1/\tau_i$ is a function of $\delta_0 = E/E_t$ (E_t being the REL avalanche threshold field) and is $\propto N_m$, the atmospheric molecular density dependent on altitude [*Gurevich et al.*, 1992, 1994; *Roussel-Dupré et al.*, 1994]. The rate $1/\tau_i > 0$ for $\delta_0 \geq 1.5$ and $= 0$ for $E_t < E < 1.5E_t$. For $E < E_t$, the REL avalanche is no longer possible, and the former REL will thermalize. In this case we take [*Lehtinen et al.*, 1996b]: $1/\tau_i = -v_R(F_D(\mathcal{E}_0) - eE)/\mathcal{E}_0 < 0$, where F_D is the dynamic friction force [*Roussel-Dupré et al.*, 1994] and $\mathcal{E}_0$ is a typical REL energy ~ 1 MeV. $\vec{v}_R$ is calculated using the relativistic equation of motion of the "average" runaway electron [*Gurevich et al.*, 1996]:

$$\frac{d\vec{p}}{dt} = -e\vec{E} - \frac{e}{m}\sqrt{1 - \frac{v_R^2}{c^2}}\left[\vec{p} \times \vec{B}_0\right] - \nu_R\vec{p}, \qquad (4)$$

where ν_R is the effective collision frequency.

Most of the free secondary electrons produced as a result of impact ionization of neutrals by REL do not become REL themselves, but instead contribute to the local ionization of the medium. The equation for the total number density of thermal electrons N_e (both ambient ionospheric and produced by REL) is:

$$\frac{\partial N_e}{\partial t} = (\nu_i - \nu_a)N_e + 2N_m v_R N_R \chi_{i0}, \qquad (5)$$

where ν_i and ν_a are the respective ionization and attachment coefficients [I], and $\chi_{i0} = 2.3 \times 10^{-22}\,\mathrm{m}^{-2}$ is the total ionization cross-section by the REL for N or O atoms [*Bell et al.*, 1995]. The second term on the RHS of (5) is overestimated by $\sim 1\%$ since it includes high energy secondaries which will become REL eventually. However, this approximation significantly simplifies the calculations.

We solve (3) in two dimensions using the modified upwind differencing scheme [*Press et al.*, 1992, p. 832]. We calculate N_R at the current time step using its value at the point in space where the particle was located at time $(t - \Delta t)$. The values between the grid points are obtained by bilinear interpolation. The von Neumann stability analysis [*Press et al.*, 1992, p. 827] shows that this method is unconditionally stable. Note that the error will be unacceptable for a mesh size $\gtrsim c\tau_i$, if one solves (3) directly for N_R. This condition can be avoided and error reduced by solving (3) for $\ln N_R$, which is analogous to an analytical solution with constant τ_i and S_o.

Optical and γ-Ray Emissions

The optical emissions are produced as a result of excitation of neutral species through impacts by: 1) the thermal electrons, N_e, driven by the QE field [I]; 2) the suprathermal electrons ($\gtrsim 10$ eV) created by the REL [*Bell et al.*, 1995]. In this paper, we calculate optical emissions associated with the first positive N_2 band, using equations (6) and (7) of [*Bell et al.* 1995].

The γ-ray emissions are produced as a result of bremsstrahlung by REL in the atmosphere, and are calculated using the technique outlined in [*Lehtinen et al.* 1996a]. To simplify calculations, we use the following facts: 1) the radiating region can be considered as a point source when viewed from a satellite; 2) almost all radiating REL have upward velocities—in fact, the contribution of REL with more than 5° deflection from the vertical is $\sim 10^{-10}$ of the total; 3) the bremsstrahlung emission is mostly in the forward direction. The angular spread of γ-rays is almost entirely due to the shape of REL momentum distribution function [*Roussel-Dupré et al.*, 1994; *Lehtinen et al.*, 1996a].

Results and Discussion

Runaway Electrons

From equation (4), for steady motion in locally homogenious $\vec{E} = E_r\hat{r} + E_z\hat{z}$ we have $v_{Rr}/v_{Rz} = (E_r/E_z)(1 + \omega_{HR}^2/\nu_R^2)^{-1}$, where ω_{HR} is the REL gyrofrequency. Therefore, REL move along $\vec{B}_0$ where $\nu_R \ll \omega_{HR}$ and along $\vec{E}$ at heights where ν_R dominates. We take $\nu_R = F_D/p \approx$ the inelastic collision rate, and find $\omega_{HR} > \nu_R$ for altitudes > 20 km, consistent with the estimate by *Gurevich et al.* [1996]. *Roussel-Dupré and Gurevich* [1996] take $\nu_R \approx$ elastic collision rate and finds $\omega_{HR} > \nu_R$ at altitudes above ~ 40 km. However since in our model the number of REL is insignificant below 40 km, the REL in the region of interest move along $\vec{B}_0$ for either value of ν_R.

Figure 1 shows the spatial distribution of REL. N_R increases very rapidly at ~ 80 km altitude over a time span of ~ 0.3 ms. However, this rapid increase in J_R does not produce any significant electromagnetic radiation, estimated to be $E_{\mathrm{rad}} \lesssim 0.15$ V/m at ~ 80 km, ~ 10 times smaller than the QE field, confirming the validity of the QE approximation. Note that this is also substantially smaller than the peak intensity of the electromagnetic pulses produced by the lightning discharges, which are of the order ~ 10 V/m at ~ 80 km altitude and which produce brief optical flashes known as "elves" [*Inan et al.*, 1996a].

Large exponential growth of REL during the avalanche causes N_R to change dramatically with the starting point altitude. N_R decreases rapidly with decreasing Q, since the avalanche starts at higher altitudes for lower values of Q. Thus, while the number of REL for $Q = 225$ C is large, they are insignificant for $Q = 150$ C (Figure 2). The post-discharge E is determined predominantly by the negative screening space charge, whose shape and altitude are influenced by value of Q, charge position and the conductivity profile.

Gurevich et al. [1994] showed that there is a transverse diffusion in the REL beam due to ionizing collisions. Using equations (26) and (31) and Figure 6 from *Gurevich et al.* [1994], we find that the beam broadening due to this kind of diffusion is $\lesssim 1$ km. This is small compared to the broadening due to the fact that the avalanche is started not by a

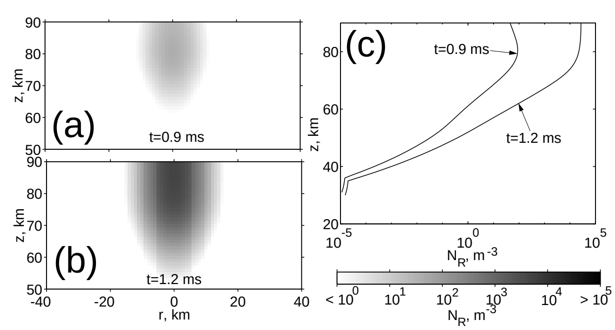

Figure 1. The spatial distribution of the REL ($Q = 225$ C, $\tau_s = 1$ ms): (a) at $t = 0.9$ ms after the start of the discharge; (b) at $t = 1.2$ ms after the start of the discharge (maximum of REL density); (c) on the axis.

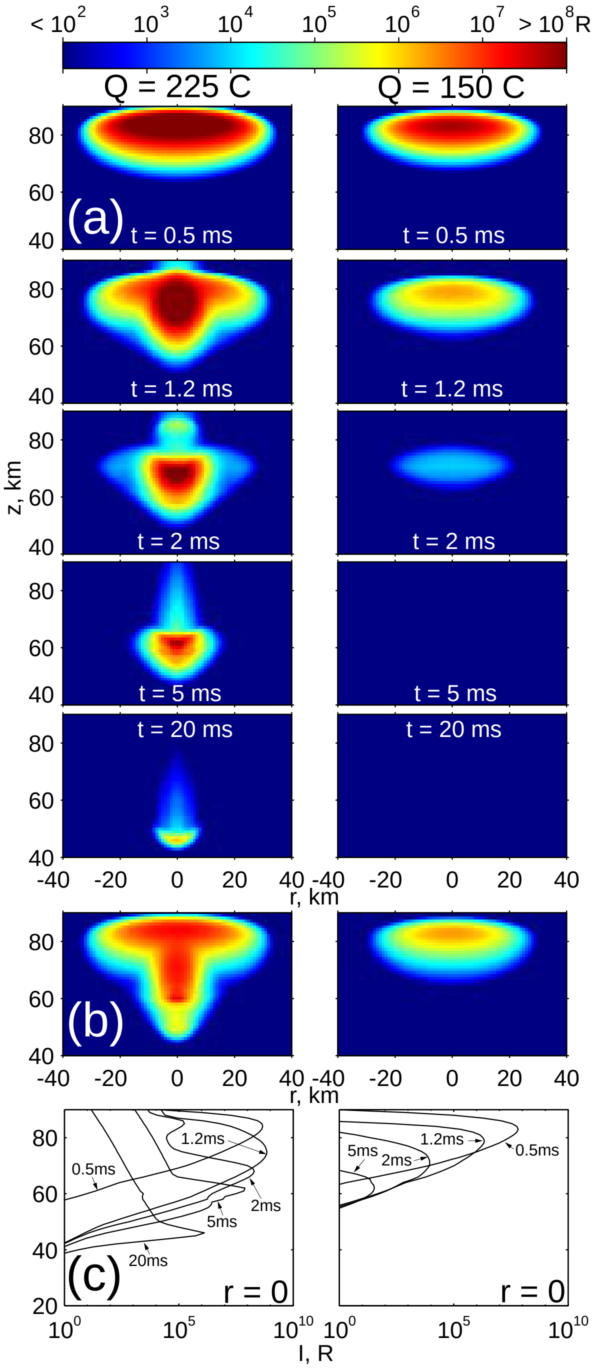

Figure 2. The 2D structure of the intensity of the N_2 first positive band at different instants of time after the beginning of the discharge, in Rayleighs, $Q = 225$ C (left) and $Q = 150$ C (right), $\tau_s = 1$ ms: (a) at different instants of time; (b) integrated over 16 ms (approximately one frame of a videocamera); (c) on the axis.

single electron, but by many electrons at different horizontal distances from the axis, which gives $\approx$ 10–15 km horizontal diameter of the beam at the 80 km altitude, defined at the level of $1/e$ of the value on the axis. This justifies the use of a fluid equation (3).

The REL create a vertical ionization channel at heights $\gtrsim$ 55 km with the same horizontal scale as the REL beam ($\sim$ 10 km). This changes the structure of E (see Figure 3): large gradients of conductivity at the narrow bottom part of the channel produce high values of ρ and E below it. The channel propagates down to $\sim$ 40 km at later times (20–30 ms) due to combined runaway and conventional ionization breakdown.

Optical and γ-Ray Emissions

The optical emission caused by suprathermal REL secondary electrons is generally much smaller than the emission due to heated thermal electrons at all altitudes and all times for a discharge of $Q = 225$ C. Figure 2 presents the intensity of the N_2 first positive band for $Q = 225$ C and $Q = 150$ C. The case of a smaller discharge gives shorter emissions that have no columnar structure at altitudes of 60–80 km, because the QE field is insufficient to initiate REL. The earlier and wider part at altitudes 80–90 km is due entirely to the QE heating of ambient electrons. Other optical bands give smaller emissions [I]. The optical emission values and geometry are consistent with the characteristics of sprites, in particular the carrot-shaped, columnar types [e.g., *Lyons*, 1996; and references therein]. The long duration ($\sim$ 20–30 ms) is caused by persisting high E at the lower boundary of the high-conductivity channel. This occurs even for a short +CG discharge ($\tau_s = 1$ ms in our case) and is consistent with observations of sprites [*Fukunishi et al.*, 1996, referenced in *Lyons*, 1996].

The predicted γ-ray fluxes (Figure 4) are consistent with the 1D model for similar discharge values [*Lehtinen et al.*, 1996a]. We see that to explain BATSE observations for 100–300 keV photons [*Fishman at al.*, 1994], a discharge $\gtrsim$ 240 C is needed. Panel (b) shows that γ-ray flux values are very sensitive to the changes in Q [see also Figure 6 in *Lehtinen et al.*, 1996a]. A significant fraction (4 out of 12 presented by *Fishman et al.* [1994]) of TGF have a duration of 3–

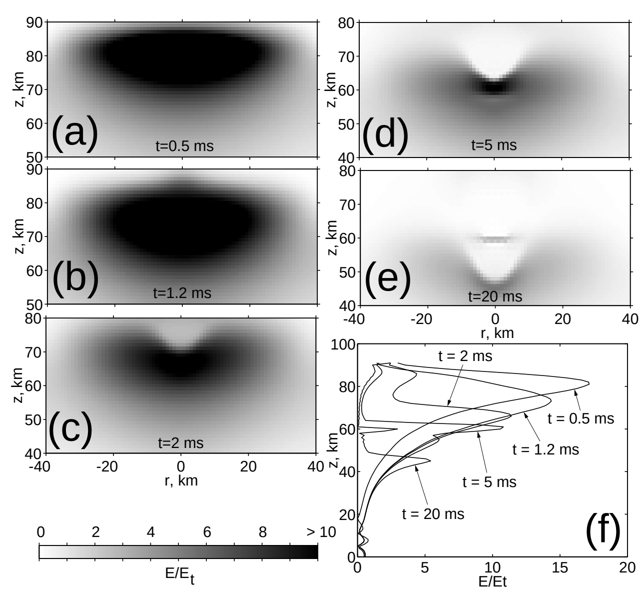

Figure 3. The ratio of the electric field to the runaway threshold field $\delta = E/E_t$ at different moments after the beginning of the discharge ($Q = 225$ C, $\tau_s = 1$ ms): (a) $t = 0.5$ ms; (b) $t = 1.2$ ms; (c) $t = 2$ ms; (d) $t = 5$ ms; (e) $t = 20$ ms; (f) on the axis.

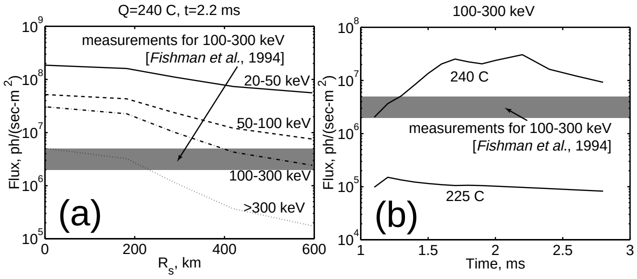

Figure 4. REL γ-ray emission fluxes at the satellite height of 500 km, integrated over BATSE energy ranges ($\tau_s = 1$ ms): (a) dependence on horizontal range R_s from the beam location at the moment of maximum emission $t = 2.2$ ms for $Q = 240$ C; (b) flux exactly above the source ($R_s = 0$) as a function of time for $Q = 225$ C and $Q = 240$ C.

5 ms, which agrees with our calculations. Our model predicts lower γ-ray fluxes at higher photon energies $\mathcal{E}_{ph}$. The spectral analysis of BATSE data [*Nemiroff et al.*, 1997] gives a photon flux dependence $\propto \mathcal{E}_{ph}^{\alpha}$ with $\alpha = -1.5$ to $-.6$, which includes the typical bremsstrahlung value $\alpha = -1$.

In comparison with the BATSE obsevations in the two cases analyzed by *Inan et al.* [1996b] it is important to take into account the direction of $\vec{B}_0$. According to Gurevich et al. [1996], in strong magnetic fields ($B_0 > E/c$) the effective accelerating field has the value $E\cos(\vec{E}, \vec{B}_0)$. Our estimate for the dip angle for the two cases analysed by *Inan et al.* [1996b] gives $\sim$ 30–35°. The effective accelerating field is $\sim 0.5E$, and to create a sufficient γ-ray flux we need to double E. It can be done either by increasing Q or by raising h_+ (the altitude of $+Q$), e.g. take $Q = 320\ C$ at $h_+ = 15$ km instead of our parameters $Q = 240$ C, $h_+ = 10$ km.

The number of REL and γ-ray emissions calculated by *Roussel-Dupré and Gurevich* [1996] and in a 2D model by *Taranenko and Roussel-Dupré* [1996] are roughly the same as in our model, but there is an important difference: the value of h_+. In the *Roussel-Dupré and Gurevich* model $Q = 100$ C and $h_+ = 18$ km. Because of the high h_+, the E field at 25 km is enhanced and the REL start at roughly this altitude. Similarly in the *Taranenko and Roussel-Dupré* model $Q = 100$ C and $h_+ = 15$ km altitude. The ratio $E/E_t \gtrsim 7$ at 20–25 km, and the REL discharge starts near 25 km. In our model, $E > E_t$ only above 35 km, and the REL avalanche over a short distance. Thus we need a larger Q to arrive at similar production of REL and associated γ-rays. Our choice of $h_+ = 10$ km is based on recent evidence that the positive discharges associated with sprites appear to originate at altitude of 10 km or lower [*Marshall et al.*, 1996].

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References

Bell, T. F., V. P. Pasko, and U. S. Inan, Runaway electrons as a source of red sprites in the mesosphere, *Geophys. Res. Lett., 22*, 2127, 1995.

Dejnakarintra, M., and C. G. Park, Lightning-induced electrical fields in the ionosphere, *J. Geophys. Res., 79*, 1903, 1974.

Fishman, G. J., P. N. Bhat, R. Malozzi, J. M. Horack, T. Koshut, C. Kouveliotou, G. N. Pendleton, C. A. Meegan, R. B. Wilson, W. S. Paciesas, S. J. Goodman, H. J. Christian, Discovery of intense gamma-ray flashes of atmospheric origin, *Science, 264*, 1313, 1994.

Gurevich, A. V., G. M. Milikh and R. Roussel-Dupré, Runaway electron mechanism of air breakdown and preconditioning during a thunderstorm, *Phys. Lett. A, 165*, 463, 1992.

Gurevich, A. V., G. M. Milikh and R. A. Roussel-Dupré, Nonuniform air-breakdown, *Phys. Lett. A, 187*, 197, 1994.

Gurevich, A. V., J. A. Valdivia, G. M. Milikh, and K. Papadopulous, Runaway electrons in the atmosphere in the presence of a magnetic field, *Radio Science, 31*, 1541, 1996.

Inan, U. S., W. A. Sampson, and Yu. N. Taranenko, Space-time structure of optical flashes and ionizaion changes produced by lightning-EMP, *Geophys. Res. Lett., 23*, 133, 1996a.

Inan, U. S., S. C. Reising, G. J. Fishman and J. M. Horack, On the association of terrestrial gamma-ray bursts with lightning and implication for sprites, *Geophys. Res. Lett., 23*, 1017, 1996b.

Lehtinen, N. G., M. Walt, U. S. Inan, T. F. Bell and V. P. Pasko, γ-ray emission produced by a relativistic beam of runaway electrons accelerated by quasi-electrostatic thundercloud fields, *Geophys. Res. Lett., 23*, 2645, 1996a.

Lehtinen, N. G., T. F. Bell, V. P. Pasko and U. S. Inan, A Two-Dimensional Model of Runaway Electron Beams Driven By Quasi-Electrostatic Thundercloud Fields, *Eos, Transactions, AGU, 77*, no. 46, F70, A11A-16, 1996b.

Lyons, W. A., Sprite observations above the U.S. High Plains in relation to their parent thunderstorm systems, *J. Geophys. Res., 101*, 29641, 1996.

Marshall, T. C., M. Stolzenburg and W. D. Rust, Electric field measurements above mesoscale convective systems, *J. Geophys. Res., 101*, 6979, 1996.

Nemiroff, R. J., J. T. Bonnell and J. P. Norris, Temporal and spectral characteristics of terrestrial gamma flashes, *J. Geophys. Res., 102*, 9659, 1997.

Pasko, V. P., U. S. Inan, T. F. Bell and Yu. N. Taranenko, Sprites Produced by Quasi-Electrostatic Heating and Ionization in the Lower Atmosphere, *J. Geophys. Res., 102*, 4529, 1997a.

Pasko, V. P., U. S. Inan and T. F. Bell, Ionospheric Effects due to Electrostatic Thundercloud Fields, *J. Atmos. Terr. Phys., in review*, 1997b.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, *Numerical Recipes in FORTRAN, 2nd ed.*, Cambridge Univ. Press, 1992.

Roussel-Dupré, R. A., A. V. Gurevich, T. Tunnel and G. M. Milikh, Kinetic theory of runaway breakdown, *Phys. Rev., 49*, 2257, 1994.

Roussel-Dupré, R. A., and A. V. Gurevich, On runaway breakdown and upward propagating discharges, *J. Geophys. Res., 101*, 2297, 1996.

Taranenko, Yu, and R. Roussel-Dupré, High altitude discharges and gamma-ray flashes: a manifestation of runaway air breakdown, *Geophys. Res. Lett., 23*, 571, 1996.

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