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Mesospheric Electric Field Transients due to Tropospheric Lightning Discharges

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Abstract. A physical picture and quantitative two-dimensional electromagnetic modeling of mesospheric electric field transients produced by cloud-to-ground (CG) lightning discharges with short duration currents (<0.5 ms) are presented. The range of applicability of existing quasi-electrostatic models of sprites and the physical conditions under which relatively weak CG lightning discharges (thundercloud charge moment changes less than 50C×10 km) may initiate sprites are discussed in the context of recent experimental findings.

Introduction

The high altitude optical emissions known as sprites [e.g., *Sentman et al.,* 1995] are believed to be produced by large electromagnetic fields capable of initiating breakdown ionization at mesospheric altitudes above thunderstorms following intense lightning discharges [e.g., *Rowland,* 1998, and references therein]. It was proposed that on time scales $>\sim$1 ms these fields are dominated by quasi-electrostatic (QE) component [e.g., *Pasko et al.,* 1997] and the dynamics of rearrangement of the current and charge systems in the conducting atmosphere above a thunderstorm, in response to the QE fields, can be understood in terms of a "moving capacitor plate" model defining a downward moving boundary which separates regions of the atmosphere dominated by the conduction (above) and displacement (below) currents [*Greifinger and Greifinger*, 1976; *Hale and Baginski*, 1987; *Hale*, 1994; *Pasko et al.,* 1998a].

The QE formulation, however, ignores retardation effects and is expected to be less applicable on small time scales (<1 ms) comparable to the speed of light travel time between the ground and the lower ionosphere. Indeed, the establishment of the QE field in response to any 'sudden' (i.e., fast, on time scale $\ll$1 ms) change in the thundercloud charge (i.e., deposition or removal of charge by a lightning current at altitude h_Q) occurs by means of waves of potential traveling at the speed of light (c) which reflect back and forth between the ground and the ionosphere. The back and forth propagation is similar to propagation and reflection of transient disturbances on a short circuited electrical transmission line [e.g., see Chapter 2 of *Inan and Inan*, 1998]. In our context, each reflection corresponds to the formation of an image charge in the conductor. Since the total electric (E) field of the deposited (removed) thundercloud charge and its image in the ionosphere is significantly greater at mesospheric altitudes than the field involving the ground image of the thundercloud charge, one would expect a transient enhancement of the field at mesospheric altitudes in comparison with QE solutions produced due to the time delay $2h_Q/c$ corresponding to the round trip between the cloud and the ground.

The purpose of this paper is to provide a description of the mesospheric E field transients associated with the speed of light travel time delay between the cloud and the ground, to assess the range of applicability of the QE formulation used in previous work and to discuss the physical conditions under which relatively weak CG lightning discharges with small vertical charge-moment changes (<50C×10 km [*Cummer and Inan*, 1997] and [*Bell et al.*, 1998]) may initiate sprites.

Submillisecond Electric Field Transients

To provide physical insight into the effect of the mesospheric E field transients, we start by considering a simplified one-dimensional (1-D) problem allowing analytical solutions. We then proceed to more general modeling of the effect using a two-dimensional (2-D) electromagnetic model.

The electric breakdown of air associated with sprites starts at the altitudes where the E field exceeds the breakdown threshold (E_k) (Fig. 1a). Fig. 1a shows an altitude scan of the E field created by a static charge Q=1000 C of various geometries placed at h_Q=10 km altitude in free space between two perfectly conducting planes at the ground and the ionosphere (h_t=95 km), both assumed to be maintained at zero potential (φ=0). The figure illustrates the maximum possible magnitude of the postdischarge QE field which is not distorted by the ambient atmospheric conductivity. This distribution can be linearly scaled to evaluate fields corresponding to different Q. Fig. 1a shows that at least 1000 C of charge is required to initiate breakdown (E>E_k) at $\sim$50 km altitude while only 10 C may be sufficient at altitudes $\sim$90 km in cases of a sufficiently low ambient conductivity, allowing effective upward penetration of the E field to this altitude [e.g., *Pasko et al.,* 1997].

For the case of a planar charge distribution with surface density ρ_s placed at an altitude h_Q=10 km in a 1-D system consisting of two conducting planes with separation h_t=95 km, the electrostatic solution for the absolute value of the E field is h_Q/h_t for $h_Q<z<h_t$ and 1-h_Q/h_t for $0<z<h_Q$, where z stands for altitude, and where both the E field and potential φ hereafter are normalized by ρ_s/ε_o for the sake of brevity. This solution is illustrated in Fig. 1a and 2a (dashed line) with ρ_s evaluated assuming that Q=1000 C is distributed over a circular disk with radius R_s=50 km.

Now consider an instantaneous (i.e., 'sudden') introduction of the same plane charge at altitude z=h_Q at the moment of time t=0. The time dynamics of the E field following the introduction of the charge shows significant deviation from the static solution as illustrated in Fig. 2a

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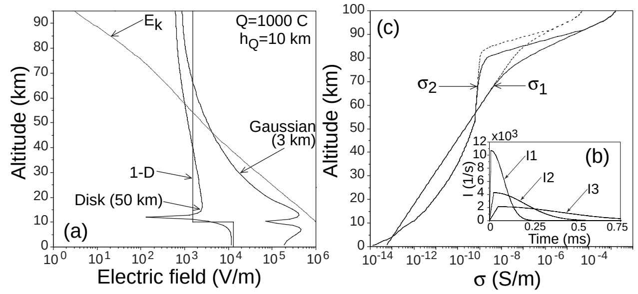

Figure 1. (a) Altitude scans at $r = 0$ of the QE field for thundercloud charge Q=1000 C of different geometries placed at 10 km altitude. The field E_k is the characteristic air breakdown field [e.g., *Papadopoulos et al.,* 1993]; (b) Three models of the lightning current used in calculations (normalized as $\int Idt = 1$). The model I1 has the linear rise time τ_r=10μs and the exponential fall time τ_f=0.1 ms; I2: τ_r=25μs and τ_f=0.25 ms; I3: τ_r=50μs and τ_f=0.50 ms; (c) Dashed lines show two altitude profiles of conductivity used in the calculations, which correspond to the ionospheric electrons heated by the E field $\simeq E_k$ (the electron mobility data is taken from *Pasko et al.,* 1997, Figure 7). Solid lines illustrate cold (ambient) profiles. Profile 1 is taken from [*Pasko et al.,* 1997] and Profile 2 from [*Hale,* 1994].

at three selected instants of time. This dynamics is described by waves of potential as illustrated in Fig. 2b,c. Initially, upward and downward moving waves are launched φ_1^+=-z/2+h_Q/2 and φ_1^-=z/2-h_Q/2, which produce reflected waves φ_2^-=$-z$/2+h_t-h_Q/2 and φ_2^+=z/2+h_Q/2, respectively. Waves φ_2^- and φ_2^+ produce secondary reflected waves φ_3^+=-z/2-h_t+h_Q/2 and φ_3^-=z/2-h_t-h_Q/2 which meet at altitude h_Q at the moment of time corresponding to the round-trip travel time between the two planes (T=2h_t/c), at which time the E field becomes zero everywhere in the system (i.e., identical to t=0). The entire sequence is repeated with period T, the static solution being never established; however, we note that the E field is equal to the static solution both above and below h_Q when averaged in time over T. Each reflection generates the oppositely traveling potential wave with opposite sign to match zero potential boundary condition on the conducting boundaries at z=0 and z=h_t, similar to reflections on a short circuited transmission line [*Inan and Inan,* 1998, Chapter 2]. In terms of the E field the reflection leads to doubling of the E field magnitude since it corresponds to the formation of image charge in a conductor. The continuous oscillations in 1-D are a consequence of the fact that all images produce the identical E field contribution no matter what are the distances of a charge and its image from the conducting boundary.

The E field magnitude corresponding to φ_1^+ is 0.5 which is a factor of 5 greater than the static solution of $h_Q/h_t \simeq$0.1 (Fig. 2a, t=(h_t-h_Q)/c). This field is doubled by the first reflection from the ionosphere giving a transient with amplitude nearly 10 times greater than the static solution (Fig. 2a, t=h_t/c). Physically, the transient is produced due to the speed of light travel time delay 2h_Q/c (i.e., the round trip time between the cloud and the ground) by which ground reflected potential φ_2^+ (effectively carrying information about the ground image charge) is delayed with respect to the upward going potential wave φ_1^+ (Fig. 2). The E field is reduced at times t>h_t/c after φ_2^- and φ_2^+ waves meet at altitude h_t-h_Q at time t=h_t/c (Fig. 2, t=2(h_t-h_Q)/c).

We note the occurrence of transient fields with amplitudes a factor of h_t/h_Q~10 greater than the QE solutions. The maximum E field after the first reflection from the upper boundary (i.e., E=1) in 1-D does not depend on h_Q and h_t and is simply the field of a plane charge placed near the conducting boundary (i.e., ionosphere). The electrostatic solution involving the ground boundary is h_t/h_Q times smaller than this transient. Thus for a charge at h_Q=10 meters, for example, the transient will be h_t/h_Q=10000 times greater than the static solution; however, its duration (~h_Q/c) will be very short. These transients may allow a significant lowering of the charge values required to produce sprites. However, source lightning discharges with finite durations should be investigated before definitive conclusions can be reached.

The above field dynamics at each altitude $E_o(z,t)$ can be considered as the impulse response of a linear time invariant system, thus allowing simple analytical calculations for the field $E(z,t)$ dynamics for any given source function $I(t)$ (Fig. 1b), as the convolution of $E_o(z,t)$ with $I(t)$. Results are shown in Fig. 3a for source functions $I1, I2$, and $I3$ (Fig. 1b). The enhancement of the field in comparison with the QE solution, shown by the dashed line, is three times at 70 km altitude and is as high as four times between 80 and 95 km for the shortest duration source $I1$. The magnitude of the transient is reduced with increasing source duration (cases $I2$ and $I3$), as expected.

Fig. 3b shows 2-D solutions for the same source functions depositing a point charge Q=1000 C at 10 km altitude. The solutions are obtained from numerical integration of the full set of Maxwell's equations using space and time centered finite differences [*Birdsall and Langdon*, 1991, p. 353]. The model treats the transverse magnetic (TM) component (E_r, E_z, B_ϕ) of the electromagnetic field in a cylindrical coordinate system (r, z, ϕ) with z axis representing altitude, and assumes azimuthal symmetry ($\partial/\partial\phi$=0) [*Pasko et al.,* 1998a; *Veronis et al.,* 1999]. The QE solution is shown in Fig. 3b by dashed lines, and corresponds to the Gaussian charge distribution whose E field distribution is shown in Fig. 1a. As in the 1-D case, the 2-D solutions for the fastest source $I1$

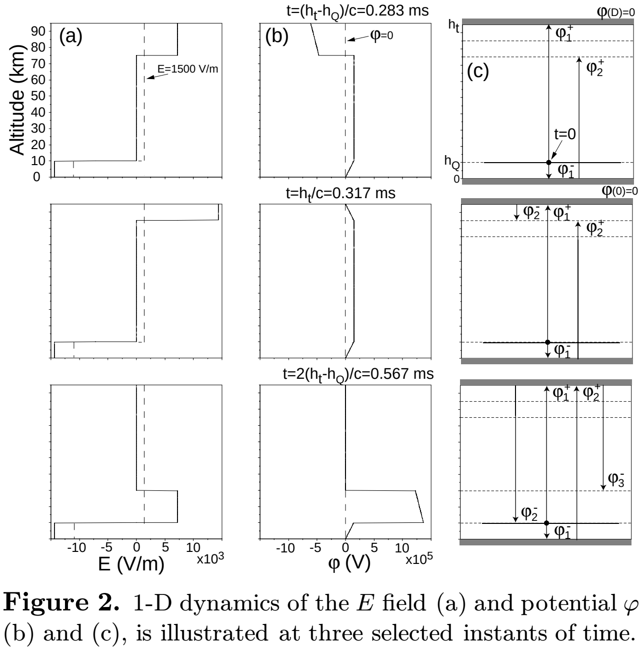

Figure 2. 1-D dynamics of the E field (a) and potential φ (b) and (c), is illustrated at three selected instants of time.

show E field oscillations in time corresponding to multiple reflections from the upper and lower boundaries. Remarkable differences from the 1-D case are associated with the fast damping of these oscillations caused by reduction of the contribution of image charges (inversely proportional to the distance squared) created by new reflections, as soon as they are formed further and further from the conducting boundaries. The E field for case $I1$ is a factor of 2 above the QE solution. In all cases all transients are over in ∼1 ms, after which time solutions are in agreement with those obtained with the QE formulation [*Pasko et al.,* 1997], as shown in Fig. 1a. For sources lasting longer than ∼0.5 ms (e.g., case $I3$) no significant transients are observed and the QE formulation can be considered valid at all times. Deviations from this rule may occur for horizontally spread charges for which geometry is closer to 1-D, as discussed above. Open crosses and circles in Fig. 3b illustrate the results of calculations for source $I1$ including realistic [*Hale*, 1994; *Pasko et al.,* 1997] ambient conductivity profiles σ_1 and σ_2, respectively, shown in Fig. 1c. The field is significantly reduced at altitudes above ∼85 km for σ_2 and above ∼70 km for σ_1. However, the initial transient peak is insensitive to conductivity and in both cases is very similar to the free space solution between two conducting planes, as expected from the fact that this transient is created by only the source charge and does not yet contain any information about image charges (i.e., conductivity profile) above 70 km, and the fact that the transient time scale (∼100μs) is much less than the dielectric relaxation time in the conducting atmosphere (ε_o/σ) everywhere below 70 km.

Discussion

To place the results of this paper in the proper perspective with respect to previous work we note that our results show the possibility of a substantial enhancement of the lightning induced QE fields associated with the speed of light travel time delay between cloud and ground. Although previously developed electromagnetic models of the lightning ionosphere interaction [e.g., *Rowland et al.,* 1996; *Fernsler and Rowland*, 1996; *Cho and Rycroft*, 1998] have included the effects of the electromagnetic fields in sprite generation none of the previous work provided a discussion of the importance of the speed of light travel time delay between cloud and ground on the establishment of the QE fields and the range of validity of the QE approximation in sprite modeling.

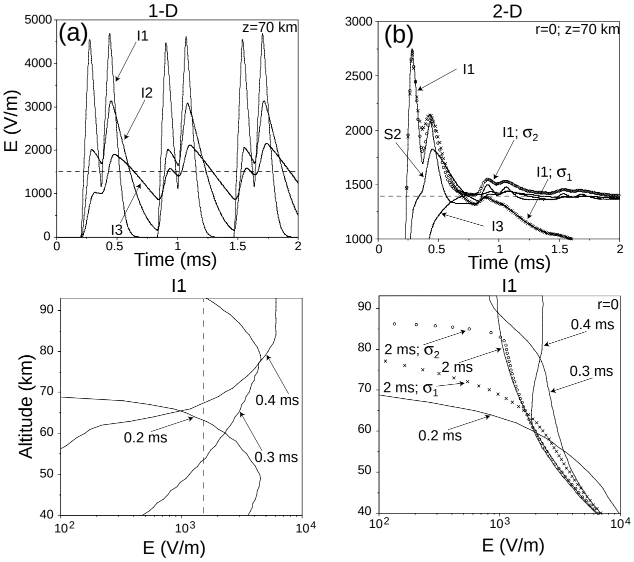

Figure 3. Dynamics of submillisecond electric field transients for different lightning current models in 1-D (a) and 2-D (b) geometries.

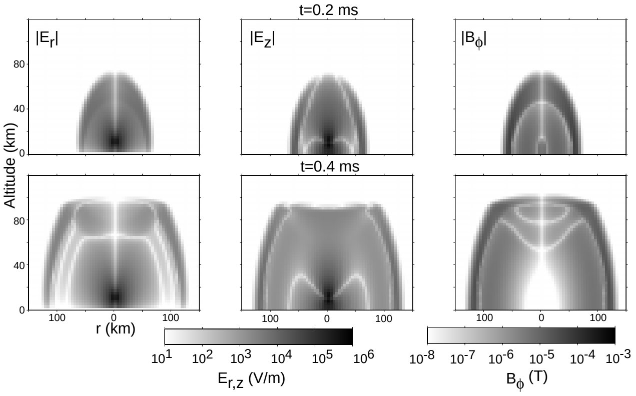

Figure 4. A cross-sectional view of the distribution of the absolute values of the TM field components (E_r, E_z, B_ϕ) at two selected instants of time.

The 1-D solution described in the previous section does not include any information about the current responsible for the deposition of the charge which then creates the vertical E field. From simple charge conservation arguments the current always must be present in a system in which the charge changes in time. If the current is vertical, it leads to the radiation of transverse magnetic and E fields which propagate simultaneously with the vertical E field. To illustrate, Fig. 4 shows 2-D distributions of E_r, E_z and B_ϕ field components at selected instants of time corresponding to the case of source $I1$ and ambient atmospheric conductivity σ_2. At the axis of symmetry (r=0) the vertical lightning current and the associated deposited charge give rise to a purely vertical E field propagating with the speed of light ($E_z \neq 0$; $E_r = 0$; $B_\phi = 0$). The TM wave packet is simultaneously launched off axis into the Earth-ionosphere waveguide for which all three field components (E_r, E_z, B_ϕ) are nonzero.

The QE field distributions shown in Fig. 1a indicate that at altitudes above ∼70 km sprites can be readily initiated ($E > E_k$) with experimentally observed charge moments of less than 100C×10 km [*Cummer and Inan,* 1997; *Bell et al.,* 1998]. The estimated charge moment of 1000C×10 km required for the breakdown at ∼50 km altitudes appears to be a factor of 2 to 3 above the largest observed charge moments [e.g., *Cummer and Inan,* 1997]. It is possible that sprites develop in two stages, the first of which is fast breakdown ($E > E_k$) corresponding to the bright upper portions of sprites, and the second is a much slower downward extension of the sprite to regions where ($E < E_k$) due to the streamer mechanism, when the E field is enhanced to $E > E_k$ only in narrow regions around the tips of highly conducting breakdown channels (streamers) [*Pasko et al.,* 1998b]. Since streamers are known to propagate in fields as low as a factor of 3 below E_k [e.g., *Grange et al.,* 1995] the breakdown ionization may extend in this form downward substantial distances. However, it is difficult to understand the observed fast formation of sprites in terms of the velocity of the propagation of streamers (∼100 km/s).

In the case of the QE model, ~10 C moved from cloud to ground in ~1 ms (10 kA current) leads to $E > E_k$ above ~90 km (see Fig. 1a). If charge is moved quickly enough (e.g., in ~100 μs) leading to the transient field enhancement, then half the amount of charge (~5 C) produces the same effect, requiring ~50 kA current. However, the ionization time scale at 90 km for $E \simeq E_k$ is ≃4 ms [e.g., *Pasko et al.,* 1997], so that only optical emissions having low thresholds (i.e., 1st positive of N_2) are effectively excited, and no significant ionization is produced. The E field should be greater than $2E_k$ to produce significant ionization at 90 km altitude on the time scale of the E field transient, or ~0.1 ms (Fig. 3b). This requires 10 C of charge moved in 0.1 ms, or a current of 100 kA, which is on the large end of currents associated with sprites [*Lyons*, 1996; *Reising et al.,* 1996]. At ~70 km altitude the ionization (as well as the optical excitation) can be easily produced for $E \simeq E_k$ by moving 50 C of charge in 0.1 ms; however, the associated current ~500 kA would then be unreasonably high. At the same time, at 70 km the QE model requires at least 100 C which, however, can be easily moved in more than ~1 ms with a reasonable current of less than ~100 kA.

The QE and electromagnetic modeling, based on CG lightning discharge, presented above in this paper have obvious difficulty to explain sprite formation at altitudes below ~50 km with measured values of vertical current and charge moments associated with sprite producing lightning discharges. It has recently been suggested by *Bell et al.* [1998] that vertical charge moments measured using ELF observations [e.g., *Cummer and Inan,* 1997] are actually only lower bounds to the total charge removed from the cloud prior to the appearance of sprites and a substantial fraction of the charge can be removed by horizontal intracloud currents. In terms of the general physical picture of penetration of the QE fields to mesospheric altitudes the only important parameters are the amount of charge and the altitude from which charge was removed [*Pasko et al.,* 1997], regardless of whether it is removed by CG lightning or neutralized with a horizontally distant charge of opposite sign by horizontal intracloud currents as suggested by *Bell et al.* [1998]. The effects of radiation fields from the horizontal currents on the lower ionosphere were studied by *Milikh et al.* [1995]; *Rowland et al.* [1996]; *Glukhov and Inan* [1996]; and *Valdivia et al.* [1998].

Summary

The submillisecond E field transients associated with the speed of light travel time delay between cloud and ground may lower the amount of charge required to initiate sprites. However, under realistic conditions these transients are expected to play an appreciable role only at higher altitudes ~80-90 km. In most cases where charge is moved by currents with duration >0.5ms the QE formulation provides accurate solutions, in particular, requiring at least ~10 C×10 km of charge moment change to initiate sprites at 90 km (assuming low ambient conductivity) and ~100 C×10 km at 70 km altitudes. The modeling based on experimentally measured vertical charge moment changes associated with sprite producing lightning has difficulty to explain the observed fast sprite formation at altitudes <50 km. Our results thus support earlier suggestions [*Bell et al.,* 1998] that substantial additional charge may be removed by horizontal intracloud currents.

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