EE 317
Spring 2001
Midterm Exam #1

Name: KEY
ID Number:
Section: 001

This exam consists of 5 pages.
The problems are worth a total of 100 points. The number of points you can earn on each problem is given in [brackets] next to the problem number.
For partial credit, clearly show all work.
Where appropriate, enclose final answers in boxes, and include units in all answers.
Calculators are permitted.
You are permitted to consult your textbook but not any other notes of any kind.

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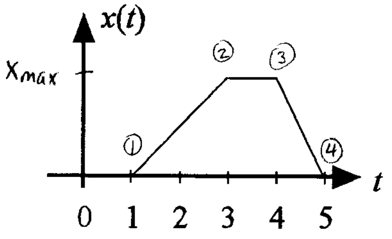
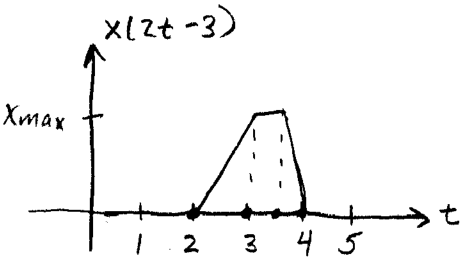
1. [0, 4, 8 points] Assuming a given signal $x(t)$ has the waveform shown below, plot $x(2t-3)$.



Let $x(\tau) = x(2t-3)$ Thus, $\tau = 2t-3 \rightarrow t = \frac{1}{2}(\tau+3)$

Feature in waveform	time τ of occurrence in $x(\tau)$	time t of occurrence in $x(2t-3)$
①	1	2
②	3	3
③	4	$7/2 = 3.5$
④	5	4



Alternative solution:

$x(2t-3) \leftarrow$ shift $x(t)$ by 3 to the right, then scale (compress in this case) by 2.

2. [2 points/property; 8 points total] The impulse response of a given LTI system is

$$h(t) = e^{-(t+3)}u(t-2) \leftarrow \text{response if } x(t) = \delta(t) \text{ (a "ping" at time } t=0)$$

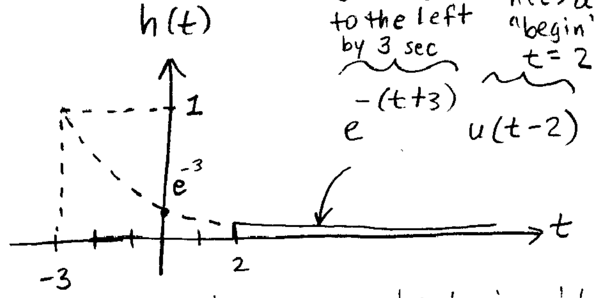
Circle the appropriate properties of the system described by this impulse response:

[✓ has memory] memoryless

[✓ causal] noncausal

[✓ BIBO stable] BIBO unstable

[✓ linear] ← LTI system! nonlinear



shift e^{-t} to the left by 3 sec

h(t) doesn't "begin" until t = 2 sec

$e^{-(t+3)} u(t-2)$

- stability: $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$?

$\int_{-\infty}^{\infty} |e^{-(t+3)}u(t-2)|\,dt = \int_{2}^{\infty} e^{-(t+3)}\,dt$

$= -e^{-(t+3)}\Big|_2^{\infty} = -e^{-\infty} + e^{-5} = e^{-5} < \infty$ ✓

1 - has memory, because output is obtained long after the "ping" at time $t=0$.

- causal, because no output before $t=0$ for a ping at $t=0$ ($h(t)=0$ for $t<0$)

3. [0, 4, 8 points] Evaluate the following integral:

use $\int_a^b f(t)\,\delta(t-t_0)\,dt = f(t_0)$, if $a \le t_0 \le b$

$$\int_0^{2\pi} \cos(kx-\omega t)\delta(t-0.5\pi)dt = \boxed{\cos\left(kx - \frac{\pi\omega}{2}\right)}$$

check: is $t = 0.5\pi$ between 0 and 2π? Yes

4. [0, 8 points] Over what range of *t* is the expression *u*(6+3*t*) - *u*(-4-5*t*) <u>non-zero</u>? (Be careful with this one! I suggest you make a sketch of each unit step function.)

a) [redacted] < t < [redacted] $\quad -2 \quad -0.8$

[✓ b)] t < [redacted], t > [redacted] $\quad -2 \quad -0.8$

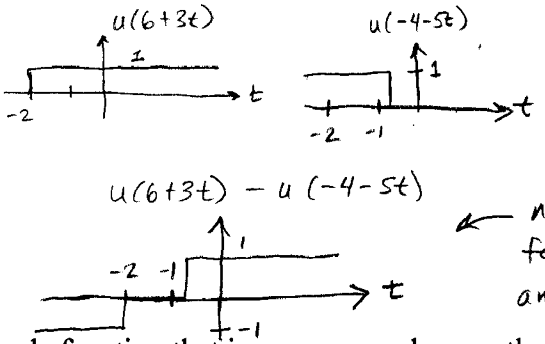
c) [redacted] < t < [redacted] 4 $\quad -6$

d) t < [redacted], t > [redacted] 4 $\quad -6$

e) the expression is zero everywhere

$u(6+3t) = 1$, if $6+3t \ge 0 \rightarrow 3t \ge -6 \rightarrow t \ge -2$

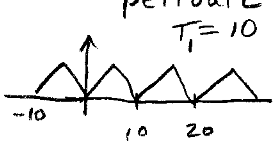
$u(-4-5t) = 1$, if $-4-5t \ge 0 \rightarrow -5t \ge 4 \rightarrow t \le -0.8$

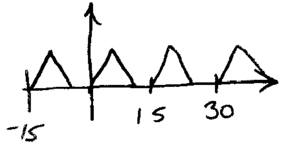


← non-zero for $t < -2$ and $t > -0.8$

5. [0, 8 points] Assume that a signal *x*(*t*) is a triangle function that is non-zero only over the range 0 < *t* < 10. Find the fundamental period of the following signal:

$$f(t) = \underbrace{\sum_{k=-\infty}^{\infty} x(t-10k)}_{\text{periodic } T_1 = 10} + \underbrace{\sum_{k=-\infty}^{\infty} x(t-15k)}_{\text{periodic, } T_2 = 15}$$





a) 5

b) 10

c) 15

[✓ d)] 30

e) 1/5

f) the signal is not periodic

The fundamental period is the least common multiple of $T_1 = 10$ and $T_2 = 15$

$\Rightarrow T_{f(t)} = 30 = 3T_1 = 2T_2$

6. [0, 8 points] The output of a certain LTI system is $y(t) = 5e^{-t}u(t-1)$ when the input signal is $x(t) = 10\,u(t)$. What is the output if the input is changed to $x(t) = 20\,u(t-2) + 5\,u(t-4)$?

[✓ a)] $y(t) = 10e^{-(t-2)}u(t-3) + 2.5e^{-(t-4)}u(t-5)$

b) $y(t) = 10e^{-t}u(t-3) + 2.5e^{-t}u(t-5)$

c) $y(t) = 10e^{-(t-2)}u(t-2) + 2.5e^{-(t-4)}u(t-4)$

d) $y(t) = 20e^{-(t-2)}u(t-3) + 5e^{-(t-4)}u(t-5)$

e) $y(t) = 20e^{-t}u(t-3) + 5e^{-t}u(t-5)$

f) $y(t) = 5e^{-t}u(t-1)$

If $y(t) = 5e^{-t}u(t-1)$ for $x(t) = 10u(t)$, then $y(t) = 0.5e^{-t}u(t-1)$ for $x(t) = u(t)$ (step response) (by linearity).

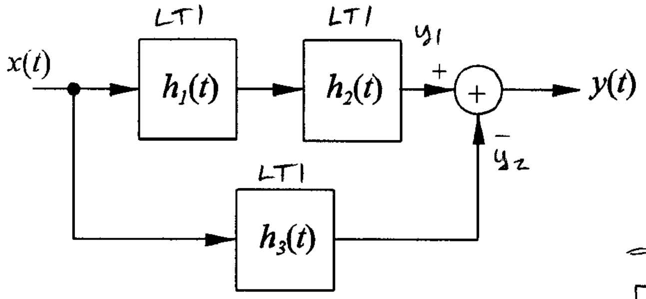
If $x(t) = 20u(t-2) + 5u(t-4)$

scale step response by 20 and shift to right by 2.

scale step response by 5 and shift to right by 4

$\rightarrow 10e^{-(t-2)}u(t-3) + 2.5e^{-(t-4)}u(t-5)$

7. [0, 4, 8 points] Assume the following block diagram represents a linear time-invariant system. What is the output $y(t)$ in terms of the input $x(t)$ and the impulse response of each subsystem?



$y(t) = y_1(t) - y_2(t)$

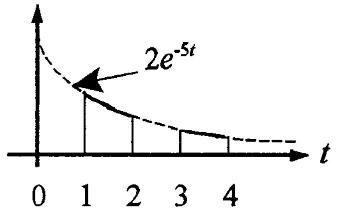
$y_1(t) = x(t) * h_1(t) * h_2(t)$

$y_2(t) = x(t) * h_3(t)$

$\Rightarrow y(t) = x(t) * h_1(t) * h_2(t) - x(t) * h_3(t)$

$$\boxed{y(t) = x(t) * [h_1(t) * h_2(t) - h_3(t)]}$$

8. [0, 8 points] Give a complete mathematical expression for the following signal. The dashed line represents the curve $2e^{-5t}$ (a guideline); however, the signal itself is represented by the solid lines.



a) $e^{-1}e^{-5t}[u(t-1) - u(t-2)] + e^{-3}e^{-5t}[u(t-3) - u(t-4)]$

b) $2e^{-5(t-1)}[u(t-1) - u(t-2) + u(t-3) - u(t-4)]$

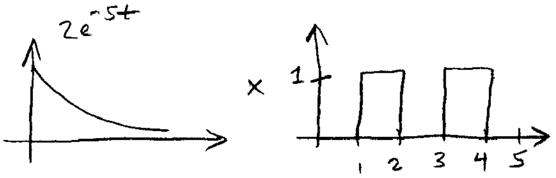
c) $2e^{-5(t-1)}u(t-1) - 2e^{-5(t-2)}u(t-2) + 2e^{-5(t-3)}u(t-3) - 2e^{-5(t-4)}u(t-4)$

[✓ d)] $2e^{-5t}[u(t-1) - u(t-2) + u(t-3) - u(t-4)]$

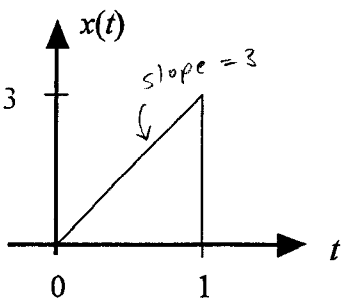
e) $2e^{-5(t-1)}u(t-1)$

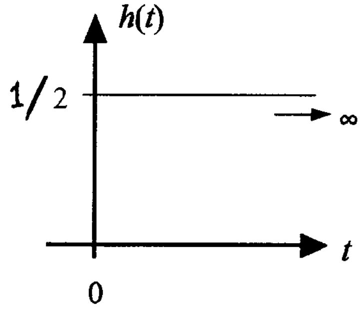
This is the function $2e^{-5t}$ multiplied by pulses between 1 and 2 seconds and between 3 and 4 seconds

i.e.,



9. [0-18 points] Using mathematical evaluation of the convolution integral, find the output *y*(*t*) of an LTI system with the following input function *x*(*t*) and impulse response *h*(*t*). Plot *y*(*t*) on the graph below.





$x(t) = 3t[u(t) - u(t-1)]$ $\qquad h(t) = \frac{1}{2}u(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} 3\tau[u(\tau)-u(\tau-1)]\frac{1}{2}u(t-\tau)\,d\tau$$

$$= \frac{3}{2}\int_{-\infty}^{\infty}\tau \underbrace{\underbrace{u(\tau)}_{=1 \text{ for } \tau>0}\underbrace{u(t-\tau)}_{=1 \text{ for } \tau<t}}_{\text{non zero for } 0<\tau<t \;\Rightarrow t>0} - \frac{3}{2}\int_{-\infty}^{\infty}\tau \underbrace{\underbrace{u(\tau-1)}_{=1 \text{ for } \tau>1}\underbrace{u(t-\tau)}_{=1 \text{ for } \tau<t}}_{\text{non zero for } 1<\tau<t \;\Rightarrow t>1}d\tau$$

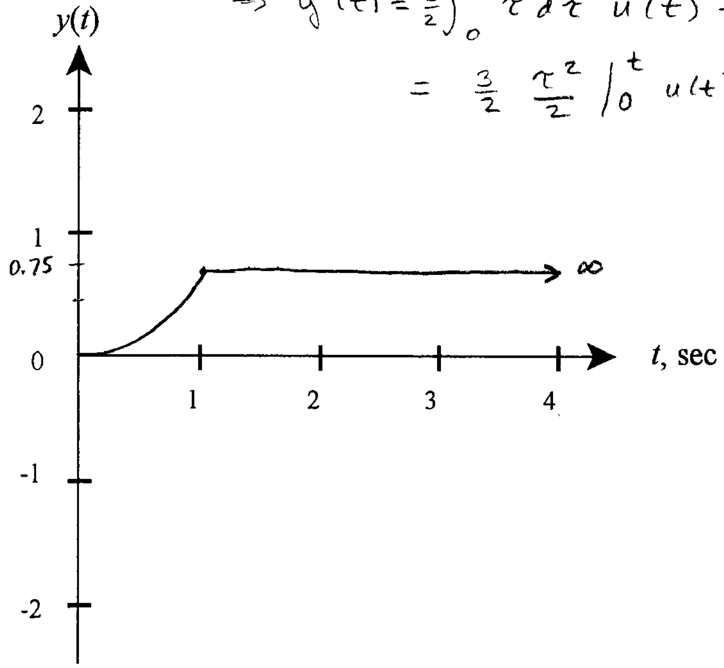
$$\Rightarrow y(t) = \frac{3}{2}\int_0^t \tau\,d\tau\;u(t) - \frac{3}{2}\int_1^t \tau\,d\tau\;u(t-1)$$

$$= \frac{3}{2}\,\frac{\tau^2}{2}\Big|_0^t u(t) - \frac{3}{2}\,\frac{\tau^2}{2}\Big|_1^t u(t-1)$$

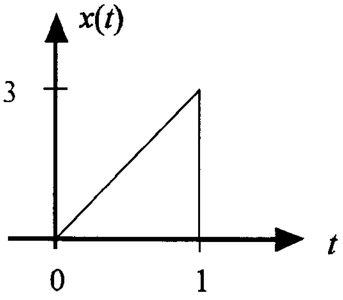
$$= \frac{3}{4}t^2 u(t) - \frac{3}{4}(t^2-1)u(t-1)$$

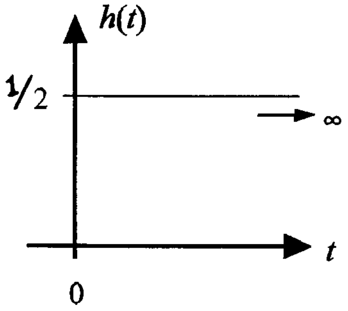
$$= \begin{cases} \frac{3}{4}t^2, & t>0 \\ \frac{3}{4}t^2 - \frac{3}{4}t^2 + \frac{3}{4}, & t>1 \end{cases}$$

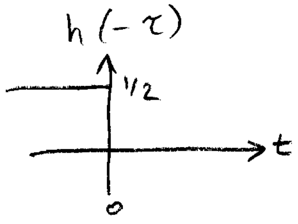
$$\Rightarrow \boxed{y(t) = \begin{cases} 0, & t<0 \\ 3t^2/4, & t \geq 0 \\ 3/4, & t \geq 1 \end{cases}}$$

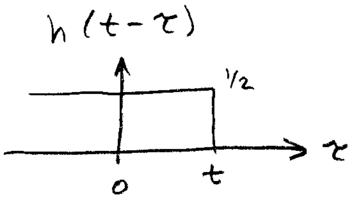


10. [0-18 points] Repeat the previous problem using graphical convolution. Plot $y(t)$ on the graph below.

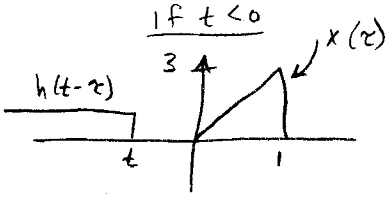








If $t<0$

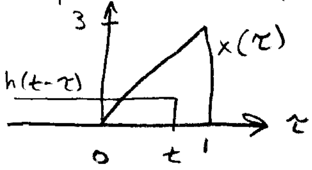


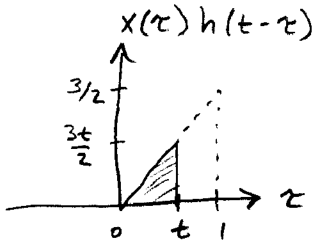
no overlap

$\Rightarrow y(t)=0$

If $0 \le t < 1$

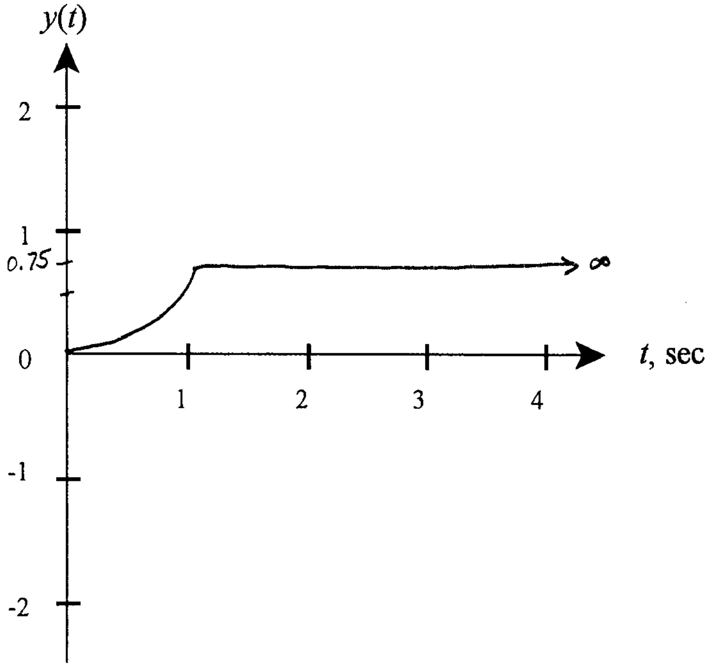
partial overlap





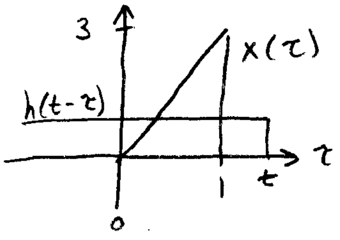
$y(t) = \text{area} = \frac{1}{2}\left(\frac{3t}{2}\right)(t)$

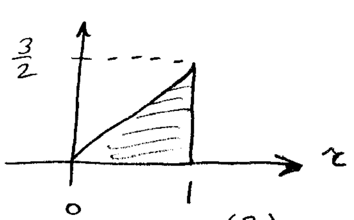
$= \frac{3}{4}t^2$



If $t \geq 1$

full overlap





$y(t) = \text{area} = \frac{1}{2}\left(\frac{3}{2}\right)(1) = \frac{3}{4}$