EE 317
Spring 2001
Final Exam

Name: KEY

ID Number: ____________________

This exam consists of 5 pages.
The problems are worth a total of 100 points. The number of points you can earn on each problem is given in [brackets] next to the problem number.
For all problems, clearly show **all** work.
Where appropriate, enclose final answers in boxes, and include units in all answers.
Calculators are permitted.
You are permitted to consult your textbook but not any other notes of any kind.

1. [1 pt/option] The impulse response of a certain discrete-time system has the following Z transform:

$$H(z) = \frac{2z^2 - 4z}{(z-1)(z+0.5)}$$

Indicate which of the following regions of convergence could possibly be applicable to this particular Z transform:

$\lvert z\rvert > 0$	possible	not possible	contains poles at $z = 1, -1/2$
$\lvert z\rvert < 0.5$, $\lvert z\rvert > 1$	possible	not possible	no value of z satisfies both constraints
$\lvert z\rvert < 0.5$	possible	not possible	no poles in ROC
$\lvert z\rvert > 0.5$, $\lvert z\rvert < 1$	possible	not possible	no poles in ROC
$\lvert z\rvert < 1$	possible	not possible	contains pole at $z = -1/2$
$\lvert z\rvert > 1$	possible	not possible	no poles in ROC

2. [0-8 points] The continuous-time signal $v(t) = 8\cos(2\pi\cdot 50t + 37°)$ is sampled at a sampling frequency of 80 Hz. The samples are then passed through an ideal reconstruction filter with a cut-off frequency of 60 Hz. What are the frequencies of the signals that pass through the reconstruction filter?

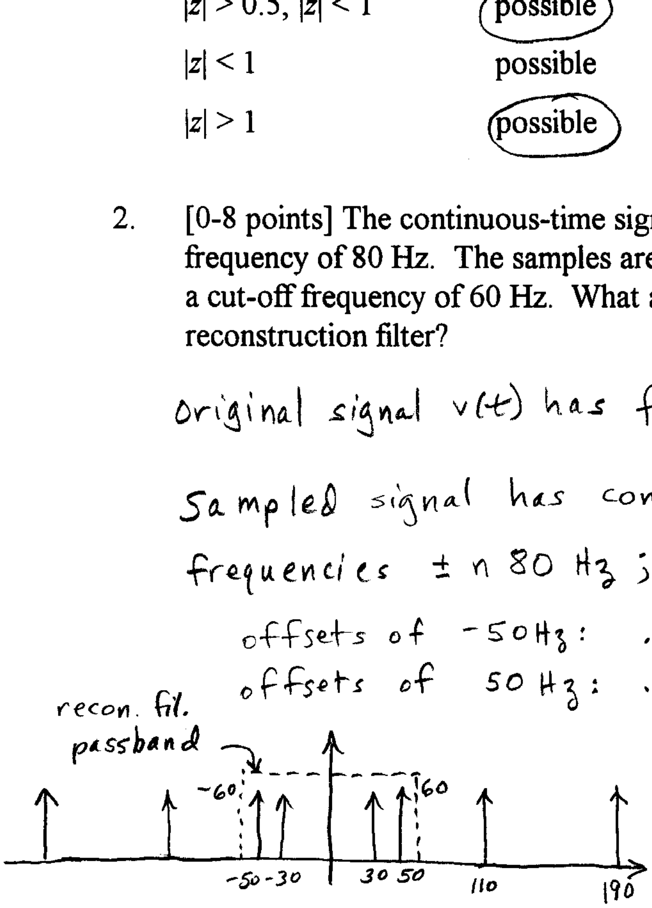
Original signal $v(t)$ has frequency components at $\pm$ 50 Hz.

Sampled signal has components at $\pm$ 50 Hz and at all frequencies $\pm n$ 80 Hz ; $n = -\infty, \ldots, 0, \ldots, \infty$

offsets of -50 Hz: $\ldots, -210, -130, -50, 30, 110, 190, \ldots$

offsets of 50 Hz: $\ldots, -190, -110, -30, 50, 130, 210, \ldots$

pass thru filter



$\boxed{\text{Passed signals are sinusoids at 30 Hz and 50 Hz}}$

3. [0, 5 points] You are designing a voice sampling system to be used in a new digital telephone network. You know that the human ear can hear sounds up to 20 kHz but that the human voice only has significant frequency components up to 3 kHz. Assuming you are only interested in transmitting voice signals through the network and that you can choose from a supply of near-perfect lowpass filters, what cut-off frequency should you use for the anti-aliasing filter in the telephone sampling system?

a) 1.5 kHz	b) 3 kHz	c) 6 kHz	d) 10 kHz
e) 20 kHz	f) 23 kHz	g) 26 kHz	h) 40 kHz

4. [0, 5 points] What minimum sampling rate should you use in the telephone system described in Problem 3? $f_s = 2 f_{max}$

a) 1.5 kHz	b) 3 kHz	c) 6 kHz	d) 10 kHz
e) 20 kHz	f) 23 kHz	g) 26 kHz	h) 40 kHz

5. [0, 5 points] Which of the following statements does **not** partially explain why it is impossible to sample a continuous-time signal and then reconstruct it perfectly in a practical digital sampling and reconstruction system?

a) It is impossible to generate a true impulse train.

b) Practical anti-aliasing filters are causal.

c) Practical reconstruction filters are causal.

d) A practical reconstruction filter passes frequencies above the Nyquist frequency.

e) In a digital system, the sample values must take one of 2^n values, where n is the number of bits in the A/D converter.

f) None of the above - they're all good explanations.

6. [0-8 points] If $f[n] = \delta[n-2]$ and $g[n] = \delta[n+1]$, find $f[n]*g[n]$ (the convolution of the two functions). You may use either a mathematical or a graphical approach.

math.

$$f[n]*g[n] = \sum_{k=-\infty}^{\infty} f[k]\,g[n-k]$$

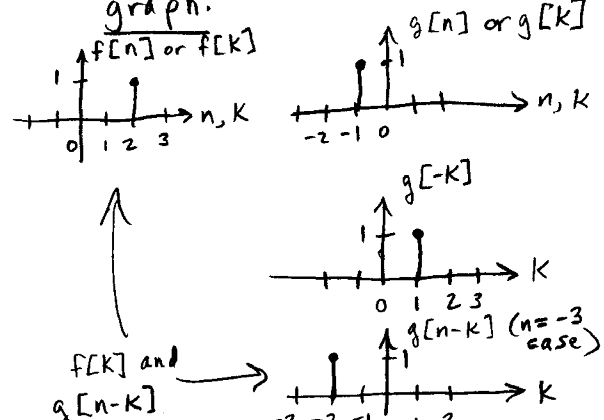
$$\delta[n-2]*\delta[n+1] = \sum_{k=-\infty}^{\infty} \delta[k-2]\,\delta[n-k+1]$$

only nonzero at $k=2$

$$\Rightarrow \delta[n-2]*\delta[n+1] = \delta[0]\,\delta[n-2+1] \quad \leftarrow k=2 \text{ term}$$

$$= \boxed{\delta[n-1]} \quad (\text{because } \delta[0]=1)$$

graph.



f[k] and g[n-k] overlap when n = 1

$\Rightarrow f[n]*g[n] = \delta[n-1]$

7. [0, 5 points] The power transfer characteristic of a continuous-time *RC* highpass filter is given by:

$$|H(\omega)|^2 = \frac{(\omega RC)^2}{1+(\omega RC)^2}$$

What is the maximum value of $|H(\omega)|^2$?

By inspection, $|H(\omega)|^2_{max}$ occurs when $\omega \to \infty$.

When $\omega \to \infty$, $|H(\omega\to\infty)|^2 = \boxed{1}$

8. [0-8 points] What is the **10-dB** frequency of the *RC* filter in Problem 7? (At what frequency does the filter pass 1/10 the maximum possible amount of power?)

$|H(\omega)|^2_{max} = 1$

Frequency at which $|H|^2 = 0.1$:

$$|H(\omega_{10})|^2 = \frac{(\omega_{10} RC)^2}{1+(\omega_{10} RC)^2} = 0.1 \longrightarrow 0.1\left[1+(\omega_{10}RC)^2\right] = (\omega_{10} RC)^2$$

$0.1 + 0.1(\omega_{10}RC)^2 = (\omega_{10}RC)^2$

$0.1 = 0.9(\omega_{10}RC)^2$

$\omega_{10}RC = \sqrt{\frac{0.1}{0.9}}$

$\boxed{\omega_{10} = \frac{1}{3}\frac{1}{RC}}$

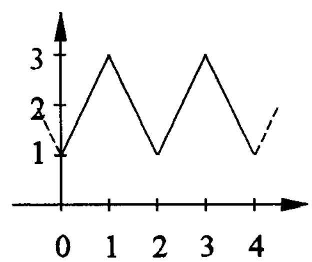
9. [0, 5 points] What is the zero-frequency or DC coefficient (C_o) in the Fourier series representation of the raised triangle wave below?

C_o = average value of sig.

By inspection,

$C_o = 2$

a) 0
b) 1
c) 2
d) 3
e) not enough information to solve



10. [0, 5 points] An input signal *x*(*t*) is applied to the input of a continuous-time LTI system. The input signal has a duration of 15 seconds. The impulse response *h*(*t*) of the system is a rectangular pulse with a width of 0.5 seconds. What is the total time duration of the output signal *y*(*t*)?

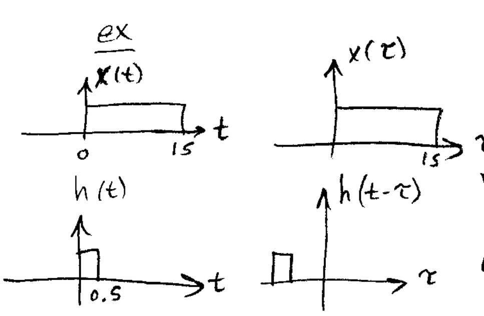
duration of $y(t)$ is the sum of the durations of $x(t)$ and $h(t)$.

a) 0 sec	b) 0.25 sec	c) 0.5 sec	d) 7.5 sec
e) 14.5 sec	f) 15 sec	g) 15.5 sec	h) ∞

$L_y = L_x + L_h = 15 + 0.5$

$= 15.5$ sec.

ex



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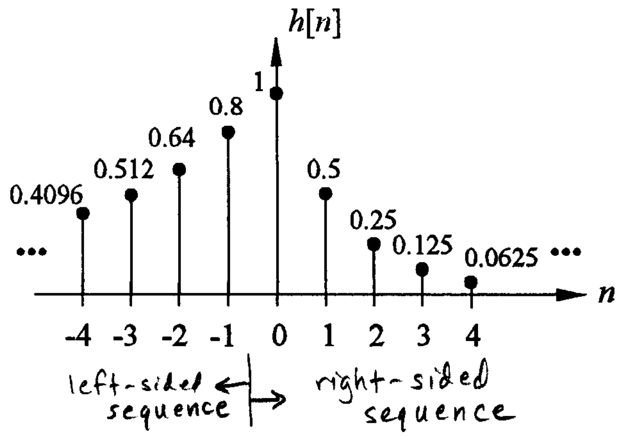
overlap occurs for $0 \le t \le 15.5 \Rightarrow y(t)$ is non-zero for 15.5 sec.

11. [0-16 points] Find the Z transform $H(z)$ for the discrete-time impulse response shown in the plot below. Although only the sequence values from $n = -4$ to 4 are shown, the sequence has non-zero values out to $n = -\infty$ for negative n and to $n = +\infty$ for positive n. (*Hint* #1: Express $h[n]$ as the sum of a left-sided signal and a right-sided signal.)



Hint #2:

$1.25^{-2} = 0.64$
$1.25^{-3} = 0.512$
$1.25^{-4} = 0.4096$
$0.5^{2} = 0.25$
$0.5^{3} = 0.125$
$0.5^{4} = 0.0625$

The sequence keeps varying as 1.25^n to $n = -\infty$ and as 0.5^n to $n = +\infty$.

$$h[n] = \underbrace{1.25^n u[-n-1]}_{\text{left-sided sequence}} + \underbrace{0.5^n u[n]}_{\text{right-sided sequence}}$$

z-transform:

$$H(z) = Z\{1.25^n u[-n-1]\} + Z\{0.5^n u[n]\}$$

$$= -Z\{-(1.25)^n u[-n-1]\} + Z\{0.5^n u[n]\}$$

$$= -\underset{\text{ROC: } |z|<1.25}{\frac{z}{z-1.25}} + \underset{\text{ROC: } |z|>0.5}{\frac{z}{z-0.5}}$$

overall ROC is the intersection of the two ROCs

$$\Rightarrow \boxed{H(z) = \frac{-z}{z-1.25} + \frac{z}{z-0.5}, \quad 0.5 < |z| < 1.25}$$

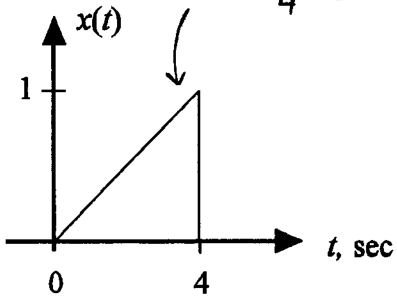
$$= \frac{-z(z-0.5) + z(z-1.25)}{(z-1.25)(z-0.5)}$$

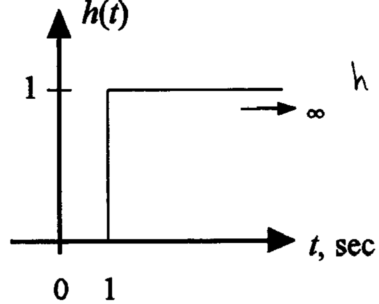
or

$$\boxed{H(z) = \frac{-0.75z}{z^2 - 1.75z + 0.625} \quad 0.5 < |z| < 1.25}$$

12. [0-16 points] Find the output $y(t)$ of the continuous-time LTI system with the following input function $x(t)$ and impulse response $h(t)$. You may use either graphical or mathematical convolution.

$x(t) = \frac{1}{4}t\left[u(t) - u(t-4)\right]$





$h(t) = u(t-1)$

math,

* graphical solution on next page.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{4}\tau\left[u(\tau) - u(\tau-4)\right] u(t-\tau-1)\, d\tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{4}\tau\, u(\tau)\, u[(t-1)-\tau]\, d\tau - \int_{-\infty}^{\infty} \frac{1}{4}\tau\, u(\tau-4)\, u[(t-1)-\tau]\, d\tau$$

$u(\tau)$: $=1$ for $\tau > 0$; $u[(t-1)-\tau]$: $=1$ for $\tau < t-1$ → must have $t-1>0$ or $t>1$

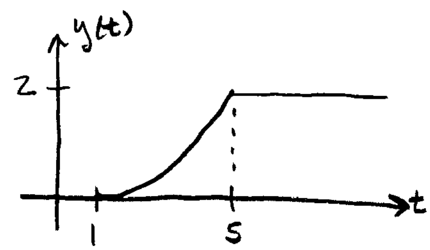
$u(\tau-4)$: $=1$ for $\tau > 4$; $u[(t-1)-\tau]$: $=1$ for $\tau < t-1$ → must have $t-1>4$ or $t>5$

$$= \int_{0}^{t-1} \frac{1}{4}\tau\, d\tau\; u(t-1) - \int_{4}^{t-1} \frac{1}{4}\tau\, d\tau\; u(t-5)$$

$$= \frac{1}{4}\cdot\frac{\tau^2}{2}\Big|_0^{t-1} u(t-1) - \frac{1}{4}\cdot\frac{\tau^2}{2}\Big|_4^{t-1} u(t-5)$$

$$= \frac{1}{8}(t-1)^2 u(t-1) - \left[\frac{1}{8}(t-1)^2 - \frac{1}{8}(16)\right] u(t-5)$$

$$= \frac{1}{8}(t-1)^2 u(t-1) - \frac{1}{8}(t-1)^2 u(t-5) + 2u(t-5) = \boxed{\begin{cases} 0, & t<1 \\ \frac{1}{8}(t-1)^2, & 1 \le t \le 5 \\ 2, & t>5 \end{cases}}$$



13. [8 points] What was your favorite part of EE 317 this semester?

a) Graphical convolution.
b) Midterm exam #2.
c) Watching Dr. Kelley try not to fall off the dais during class.
d) The aliasing videotape.
e) Other ______________________________

Have a safe and relaxing summer break!