Homework 4: Due March 24nd on Dropbox in Angel

Posted on March 10th

Collaboration policy: You are allowed to discuss homework problems with your classmates. However, you should think about the problems before discussing them, and you must write up your own solutions. You must acknowledge any collaboration by writing your collaborators names on the front page of the assignment.

Citation policy: Try to solve the problems without reading any published literature or websites, besides the textbook and notes from class. If, however, you do use a solution or part of a solution that you found in the literature or on the web, you must cite it. Furthermore, you must write up the solution in your own words.

Submission: Homeworks typeset in latex are encouraged. If you submit a hand-written homework, please ensure that it is written legibly. The homeworks must be uploaded in the dropbox provided on Angel.

1. (10 points)
   - Construct a finite field of 16 elements as $F_2(\alpha)/\alpha^4 + \alpha + 1$.
   - Show that α is a primitive element in the field, and express every element of the field as a power of α.
   - Identify all the primitive elements in the field.
   - Identify all the roots of the polynomial $x^3 - 1$.
   - Find the multiplicative inverse of $\alpha^3 + \alpha^2 + \alpha + 1$.
2. (10 points) We use the field of the previous problem to construct a conventional, primitive ($n = 15, k = 9$) Reed Solomon code.
   (a) Write the generator and parity check matrix of the Reed Solomon code. Express entries as powers of the primitive element α.
   (b) Find the generator polynomial of the code.
   (c) Write out *any* non-zero codeword of the Reed Solomon code. Erase the 11th and 12th co-ordinates. Then use Lagrange interpolation to reconstruct the erased co-ordinates. from the remaining surviving co-ordinates.
   (d) Suppose that the received symbol vector is

   $$y(x) = (0, 0, 0, \alpha^7, 0, 0, \alpha^3, 0, 0, 0, 0, 0, \alpha^4, 0, 0)$$

   Assuming that the number of errors is at most 3, find the transmitted codeword. Use any of the decoding algorithms taught in class.
3. (5 points) Problem 3.11
4. (10 points) Problem 3.31, parts 1-5. (Hint, for part 4, show that the polynomials $T_{\Phi:F}(\nu_1 x)$ and $T_{\Phi:F}(\nu_2 x)$ are distinct if $\nu_1 \neq nu_2$. To show this, use the fact that $\nu_1^q - \nu_2^q = (\nu_1 - \nu_2)^q$ for any field with characteristic q).

   As bonus a question (with no points allocated), try to solve the remainder of the problem.