Transmission Completion Time Minimization in an Energy Harvesting System

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***Abstract*—We consider the transmission completion time minimization problem in a single-user energy harvesting wireless communication system. In this system, both the data packets and the harvested energy are modelled to arrive at the source node randomly. Our goal is to adaptively change the transmission rate according to the traffic load and available energy, such that the transmission completion time is minimized. Under a deterministic system setting, we assume that the energy harvesting times and harvested energy amounts are known before the transmission starts. For the data traffic arrivals, we consider two different scenarios. In the first scenario, we assume that all bits have arrived and are ready at the transmitter before the transmission starts. In the second scenario we consider, packets arrive during the transmissions with known arriving times and sizes. We develop optimal off-line scheduling policies which minimize the overall transmission completion time under causality constraints on both data and energy arrivals.**

I. Introduction

In this work, we consider networks where nodes are able to harvest energy from nature. The nodes may harvest energy through solar cells, vibration absorption devices, water mills, thermoelectric generators, microbial fuel cells, etc. While we will not focus on how energy is harvested, we will focus on developing transmission methods that will take into account the *randomness* both in the *arrivals of the data packets* as well as in the *arrivals of harvested energy*. As shown in Fig. 1, we will consider a single node, where packets arrive at random times marked with $\times$ and energy arrives (is harvested) at random points in time marked with $\circ$. In Fig. 1, B_i denotes the number of bits in the ith arriving data packet, and E_i denotes the amount of energy in the ith energy arrival (energy harvesting). Our goal then will be to develop methods of transmission to minimize the time, T, by which all of the data packets are delivered to the destination; we call this problem *transmission completion time* minimization problem. The most challenging aspect of our optimization problem is the *causality* constraints introduced by the packet and energy arrival times, i.e., a packet may not be delivered before it has arrived and energy may not be used before it is harvested.

The trade-off relationship between delay and energy has been well investigated in traditional battery powered (unrechargeable) systems. References [1]–[6] investigate energy minimization problems with various deadline constraints. Reference [1] considers the problem of minimizing the energy in delivering all packets to the destination by a deadline. It develops a *lazy scheduling algorithm*, where the transmission times of all packets are equalized as much as possible, subject to the deadline and causality constraints, i.e, all packets must be delivered by the deadline and no packet may be transmitted before it has arrived. This algorithm also elongates the transmission time of each packet as much as possible, hence the name, *lazy* scheduling. Under a similar system setting, [2] proposes an interesting novel calculus approach to solve the energy minimization problem with individual deadlines for each packet. Reference [3] develops dynamic programming formulations and optimality conditions for a situation where channel gain varies stochastically over time. Reference [4] considers energy-efficient packet transmission with individual packet delay constraints over a fading channel, and develops a recursive algorithm to find an optimal off-line schedule. This optimal off-line scheduler equalizes the energy-rate derivative function as much as possible subject to the deadline and causality constraints. References [5] and [6] extend the single-user problem to multi-user scenarios. Under a setting similar to [5], we investigate the average delay minimization problem with a given amount of energy, and develop iterative algorithms and analytical solutions under various data arrival assumptions in [7].

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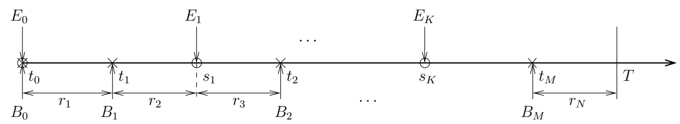

Fig. 1. System model with random packet and energy arrivals. Data packets arrive at points denoted by $\times$ and energies arrive (are harvested) at points denoted by $\circ$.

While delay minimization under a given energy constraint [7] is an important problem, it yields an intractable mathematical problem, due to varying forms the cost function takes, as a result of queueing times of previous packets affecting the delays of the future packets; see [7]. When we consider the additional energy causality constraints to be imposed in an energy harvesting system, the delay minimization problem subject to a given energy arrival profile, will become even more difficult. This becomes more evident, if one considers that, the delay minimization problem studied in [7] for a given energy constraint may be viewed as a special case of an energy harvesting system, where the energy is harvested only once at the very beginning. Consequently, in this paper, we shift our focus from delay minimization to the minimization of the *transmission completion time*, i.e., the time when the last bit is delivered to the destination. This is denoted by T in Fig. 1.

Specifically, we consider a single-user communication channel with an energy harvesting transmitter. We assume that an initial amount of energy is available at $t = 0$. As time progresses, certain amounts of energies will be harvested. In this paper, we assume that the energy harvesting procedure can be precisely predicted, i.e., that, at the beginning, we know exactly when and how much energy will be harvested. For the data arrivals, we consider two different scenarios. In the first scenario, we assume that bits have already arrived and are ready to be transmitted at the transmitter before the transmission starts. In the second scenario, we assume that packets arrive during the transmissions. However, as in the case of energy arrivals, we assume that we know exactly when and in what amounts data will arrive. Subject to the energy and data arrival constraints, our purpose is to minimize the transmission completion time of all bits through controlling the transmission rate.

This is similar to the energy minimization problem in [1], where the objective is to minimize the energy consumption with a given *deadline* constraint. In this paper, minimizing the transmission completion time is akin to minimizing the deadline in [1]. However, the problems are different, because, we do not know the exact amount of energy to be used in the transmissions, even though we know the times and amounts of harvested energy. This is because, intuitively, using more energy reduces the transmission time, however, using more energy entails waiting for energy arrivals, which increases the total transmission time. Therefore, minimizing the transmission completion time in the system requires a sophisticated utilization of the harvested energies. To that end, we develop an algorithm, which first obtains a good lower bound for the final transmission duration at the beginning, and performs energy allocation based on this lower bound. The procedure works progressively until all of the transmission rates are determined. We prove that the transmission policy obtained through this algorithm is optimal.

II. Scenario I: Bits Ready Before Transmission

We assume that there are a total of B_0 bits available at the transmitter at time $t = 0$. We also assume that there is E_0 amount of energy available at time $t = 0$, and at times s_1, s_2, ..., s_K, we have energies harvested with amounts E_1, E_2, ..., E_K, respectively. This system model is shown in Fig. 2. Our objective is to minimize the transmission completion time of these bits with these energies.

We assume that the transmitter can adaptively change its transmission power/rate according to the available energy and the remaining number of bits. We assume that the transmission rate and transmit power are related through a function, $g(p)$, i.e., $r = g(p)$. We assume that $g(p)$ satisfies the following

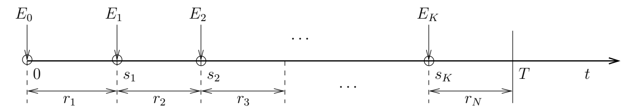

Fig. 2. System model with all bits available at the beginning. Energies arrive at points denoted by $\circ$.

properties: i) $g(0) = 0$ and $g(p) \to \infty$ as $p \to \infty$, ii) $g(p)$ increases monotonically in p, iii) $g(p)$ is strictly concave in p, iv) $g(p)$ is continuously differentiable, and v) $g(p)/p$ decrease monotonically in p. Properties i)-iii) guarantee that $g^{-1}(r)$ exists when $r \geq 0$, and that $g^{-1}(r)$ is strictly convex over the region. Property v) implies that for a fixed amount of energy, the number of bits that can be transmitted increases as the transmission duration increases. It can be verified that these properties are satisfied in many systems with realistic encoding/decoding schemes, such as optimal random coding in single-user additive white Gaussian noise channel, where $g(p) = \frac{1}{2}\log(1+p)$.

Assuming the transmitter changes its transmit power N times before it finishes the transmission, denote the sequence of transmission powers as $p_1, p_2, \ldots, p_N$, and the corresponding transmission durations of each power as $l_1,\ l_2,\ \ldots,\ l_N$, respectively. Then, the energy consumed up to time t, denoted as $E(t)$, and the total number of bits departed up to time t, denoted as $B(t)$, can be related through function g as follows:

$$E(t) = \sum_{i=1}^{\bar{i}} p_i l_i + p_{\bar{i}+1}\left(t - \sum_{i=1}^{\bar{i}} l_i\right) \tag{1}$$

$$B(t) = \sum_{i=1}^{\bar{i}} g(p_i) l_i + g(p_{\bar{i}+1})\left(t - \sum_{i=1}^{\bar{i}} l_i\right) \tag{2}$$

where $\bar{i} = \max\{i : \sum_{j=1}^{i} l_j \leq t\}$. Then, the transmission completion time minimization can be formulated as:

$$\begin{aligned} \min_{\mathbf{p},\mathbf{l}} \quad & T \\ \text{s.t.} \quad & E(t) \leq \sum_{i:t_i<t} E_i, \qquad 0 \leq t \leq T \\ & B(T) = B_0 \end{aligned} \tag{3}$$

First, we will determine some properties of the optimum solution, in the following three lemmas.

Lemma 1 *Under the optimal solution, the transmit powers increase monotonically, i.e.,* $p_1 \leq p_2 \leq \cdots \leq p_N$.

Proof: Assume that the powers do not increase monotonically, i.e., that we can find two powers such that $p_i > p_{i+1}$. The total energy consumed over this duration is $p_i l_i + p_{i+1} l_{i+1}$. Let

$$p_i' = p_{i+1}' = \frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}, \quad r_i' = r_{i+1}' = g(p_i') \tag{4}$$

Then, we have $p_i' \leq p_i$, $p_{i+1}' \geq p_{i+1}$. Since $p_i' l_i \leq p_i l_i$, the energy constraint is still satisfied, thus the new energy allocation is feasible. We use r_i', r_{i+1}' to replace r_i, r_{i+1} in the transmission policy, and keep the rest of the rates the same.

Then, the total number of bits transmitted over the duration $l_i + l_{i+1}$ becomes

$$\begin{aligned} & r_i' l_i + r_{i+1}' l_{i+1} \\ & = g\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right)(l_i + l_{i+1}) \\ & \geq g(p_i)\frac{l_i}{l_i + l_{i+1}}(l_i + l_{i+1}) + g(p_{i+1})\frac{l_{i+1}}{l_i + l_{i+1}}(l_i + l_{i+1}) \\ & = r_i l_i + r_{i+1} l_{i+1} \end{aligned} \tag{5}$$

where the inequality follows from the fact that $g(p)$ is concave in p. Therefore, the new policy departs more bits by time $\sum_{j=1}^{i+1} l_j$. Keeping the remaining transmission rates the same, the new policy will finish the entire transmission over a shorter duration. Thus, the original policy could not be optimal. Therefore, the optimal policy must have monotonically increasing rates (and powers). ■

Lemma 2 *The transmission power/rate remains constant between energy harvests, i.e., the power/rate only potentially changes when new energy arrives.*

Proof: Assume that the transmitter changes its transmission rate between two energy harvesting instances s_i, s_{i+1}. Denote the rates as r_n, r_{n+1}, and the instant when the rate changes as s_i', as in Fig. 3. Now, consider the duration $[s_i, s_{i+1})$. The total energy consumed during the duration is $p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')$. Let

$$p' = \frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}, \quad r' = g(p') \tag{6}$$

Use r' to be the new transmission rate over $[s_i, s_{i+1})$, and keep the rest of the rates the same. It is easy to check that the energy constraints are satisfied under this new policy, thus it is feasible. On the other hand, the total number of bits departed over this duration under this new policy is

$$\begin{aligned} & r'(s_{i+1} - s_i) \\ & = g\left(\frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}\right)(s_{i+1} - s_i) \\ & \geq \left(g(p_n)\frac{s_i' - s_i}{s_{i+1} - s_i} + g(p_{n+1})\frac{s_{i+1} - s_i'}{s_{i+1} - s_i}\right)(s_{i+1} - s_i) \\ & = r_n(s_i' - s_i) + r_{n+1}(s_{i+1} - s_i') \end{aligned} \tag{7}$$

where the inequality follows from the fact that $g(p)$ is a concave function of p. Therefore, the total number of bits departed under the new policy is larger than that under the original policy. If we keep all of the remaining rates the same, the transmission will be completed at an earlier time. This conflicts with the optimality of the policy. ■

Lemma 3 *Whenever the transmission rate changes, the energy consumed up to that instant equals the energy harvested up to that instant.*

Proof: From Lemma 2, we know that the transmission rate can change only at certain energy harvest instances. Assume

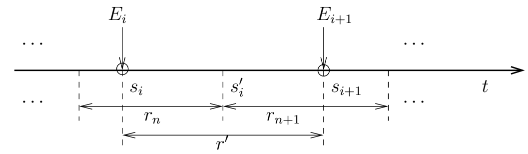

Fig. 3. The rate must remain constant between energy harvests.

that the transmission rate changes at s_i, however, the energy consumed by s_i, which is denoted as $E(s_i)$, is less than $\sum_{j=0}^{i-1} E_j$. We denote the energy gap by Δ. Denote the rates before and after t_i by r_n, r_{n+1}. Now, we can always have two small amounts of perturbations δ_n, δ_{n+1}, on the corresponding transmit powers, such that

$$p_n' = p_n + \delta_n \tag{8}$$

$$p_{n+1}' = p_{n+1} - \delta_{n+1} \tag{9}$$

$$\delta_n l_n = \delta_{n+1} l_{n+1} \tag{10}$$

We also make sure that δ is small enough such that the total energy $\delta_n l_n < \Delta$, and $p_n' \leq p_{n+1}'$. If we keep the transmission rates over the rest of the duration the same, under the new transmission policy, the energy allocation will still be feasible. The total number of bits departed over the duration $(\sum_{i=1}^{n-1} l_i, \sum_{i=1}^{n+1} l_i)$ is

$$g(p_n')l_n + g(p_n')l_{n+1} \geq g(p_n)l_n + g(p_n)l_{n+1} \tag{11}$$

where the inequality follows from the concavity of $g(p)$, and $p_n l_n + p_{n+1} l_{n+1} = p_n' l_n + p_{n+1}' l_{n+1}$, $p_n \leq p_n' \leq p_{n+1}' \leq p_{n+1}$, as in Fig. 4. This conflicts with the optimality of the original allocation. ■

We are now ready to characterize the optimum transmission policy. In order to simplify the expressions, we let $i_0 = 0$, and let $s_{m+1} = T$ if the transmission completion time T lies between s_m and s_{m+1}.

Theorem 1 *For a given B_0, and a transmission policy with power vector $\mathbf{p} = [p_1, p_2, \ldots, p_N]$ and corresponding duration vector $\mathbf{l} = [l_1, l_2, \ldots, l_N]$, the policy is optimal if and only if it has the following structure:*

$$\sum_{n=1}^{N} g(p_n) l_n = B_0 \tag{12}$$

and for $n = 1, 2, \ldots, N$,

$$i_n = \arg\min_{i: s_i \leq T} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \tag{13}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{14}$$

$$l_n = s_{i_n} - s_{i_{n-1}} \tag{15}$$

where i_n is the index of the energy arrival epoch when the power p_n switches to p_{n+1}, i.e., at $t = s_{i_n}$, p_n switches to p_{n+1}.

Proof: First, we prove that the optimal policy must have the structure given above. We prove this through contradiction.

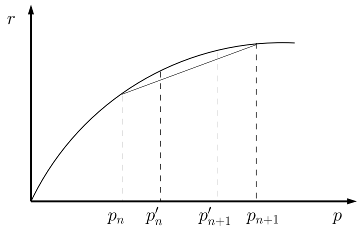

Fig. 4. $g(p)$ is concave in p.

Assume that the optimal policy, which satisfies Lemmas 1, 2 and 3, does not have the structure given above. Specifically, assume that the optimal policy over the duration $[0, s_{i_{n-1}})$ is the same as the proposed policy, however, the transmit power right after $s_{i_{n-1}}$, which is p_n, is not the smallest average power available starting from $s_{i_{n-1}}$, i.e., we can find another $s_{i'} \leq s_{i_N}$, such that

$$p_n > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} \triangleq p' \tag{16}$$

Based on Lemma 3, the energy consumed up to $s_{i_{n-1}}$ is equal to $\sum_{j=0}^{i_{n-1}-1} E_j$, i.e., there is no energy remaining at $t = s_{i_{n-1}}^-$.

We consider two possible cases here. The first case is that $s_{i'} < s_{i_n}$, as shown in Fig. 5. Under the optimal policy, the energy required to maintain a transmit power p_n over the duration $[s_{i_{n-1}}, s_{i'})$ is $p_n(s_{i'} - s_{i_{n-1}})$. Based on (16), this is greater than the total amount of energy harvested during $[s_{i_{n-1}}, s_{i'})$, which is $\sum_{j=i_{n-1}}^{i'-1} E_j$. Therefore, this energy allocation policy is infeasible. On the other hand, if $s_{i'} > s_{i_n}$, as shown in Fig. 6, then the total amount of energy harvested over $[s_{i_n}, s_{i'})$ is $\sum_{j=i_n}^{i'-1} E_j$. From (16), we know

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} > \frac{\sum_{j=i_n}^{i'-1} E_j}{s_{i'} - s_{i_n}} \tag{17}$$

Thus, under any feasible policy, there must exist a duration $l \subseteq [s_{i_n}, s_{i'})$, such that the transmit power over this duration is less than p_n. This contradicts with Lemma 1. Therefore, this policy cannot be optimal.

Next, we prove that if a policy has the structure given above, then, it must be optimal. We prove this through contradiction. We assume that there exists another policy with power vector $\mathbf{p}'$ and duration vector $\mathbf{l}'$, and the transmission completion time T' under this policy is shorter.

We assume both of the policies are the same over the duration $[0, s_{i_{n-1}})$, however, the transmit policies right after $s_{i_{n-1}}$, which are p_n and p'_n, with duration l_n and l'_n, respectively, are different. Based on the assumption, we must have $p_n < p'_n$. If $l_n < l'_n$, from Lemma 3, we know that the total energy available over $[s_{i_{n-1}}, s_{i_n})$ is equal to $p_n l_n$. Since $p_n < p'_n$, p'_n is infeasible over $[s_{i_{n-1}}, s_{i_n})$. Thus, policy $\mathbf{p}'$ could not be optimal. Then, we consider the case if $l_n > l'_n$. If $T' \geq s_{i_n}$, then, the total energy spent over $[s_{i_{n-1}}, s_{i_n})$ under $\mathbf{p}'$ is greater than $p_n l_n$, since $p'_n > p_n$, and $p'_{n+1} > p'_n$ based on Lemma 1. If $T' < s_{i_n}$, since the power-rate function g is concave, the total number of bits departed over $[s_{i_{n-1}}, s_{i_n})$ under $\mathbf{p}$ is greater than that under $\mathbf{p}'$. Thus, since policy $\mathbf{p}'$ cannot depart B_0 bits over T', it is not optimal.

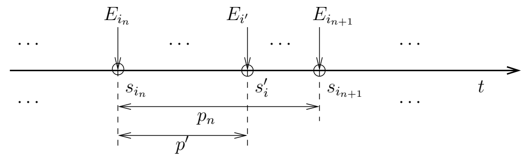

Fig. 5. Case I: $s_{i'} < s_{i_n}$.

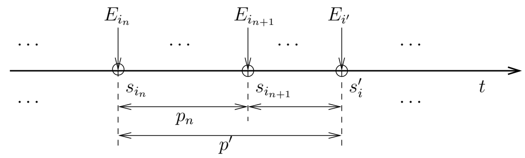

Fig. 6. Case II: $s_{i'} > s_{i_n}$.

In summary, a policy is optimal if and only if it has the structure given above. This completes the proof. ■

Therefore, we note that if the overall transmission duration T is known, then the optimal transmission policy is known via Theorem 1. On the other hand, the overall transmission time T is what we want to minimize, and we do not know its optimal value up front. Consequently, we do not know up front which energy harvests will be utilized. For example, if the number of bits is small, and E_0 is large, then, we can empty the queue before the arrival of E_1, thus, the rest of the energy arrivals are not necessary. Therefore, as a first step, we will obtain a good lower bound on the optimal transmission duration.

First, we compute the amounts of energy required to finish the entire transmission before s_1, s_2, …, s_K, respectively, at a constant rate. We denote these as A_i:

$$A_i = g^{-1}\left(\frac{B_0}{s_i}\right) s_i, \quad i = 1, 2, \ldots, K \tag{18}$$

Then, we compare A_i with $\sum_{j=0}^{i-1} E_j$, and find the smallest i such that $A_i \leq \sum_{j=0}^{i-1} E_j$. We denote this i as $\tilde{i}_1$. If no such $\tilde{i}_1$ exists, we let $\tilde{i}_1 = K + 1$.

Now, we assume that we can use $\sum_{j=0}^{\tilde{i}_1-1} E_j$ to transmit all B_0 bits at a constant rate. We allocate the energy evenly to these bits, and the transmission time T_1 is the solution of

$$g\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T}\right) T = B_0 \tag{19}$$

and the corresponding constant transmit power is

$$p_1 = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1} \tag{20}$$

Next, we compare p_1 with $\frac{\sum_{j=0}^{i-1} E_j}{s_i}$ for every $i < \tilde{i}_1$. If p_1 is smaller than every term, then, maintaining p_1 is feasible, and the optimal policy is to transmit at a constant transmission rate $g(p_1)$ with duration T_1, which gives the smallest possible transmission completion time, $i_1 = \tilde{i}_1$. Otherwise, maintaining p_1 is infeasible under the given energy arrival realization.

Thus, we update

$$i_1 = \arg\min_{i<\tilde{i}_1} \left\{ \frac{\sum_{j=0}^{i-1} E_j}{s_i} \right\}, \quad p_1 = \frac{\sum_{j=0}^{i_1-1} E_j}{s_{i_1}} \tag{21}$$

i.e., over the duration $[0, s_{i_1})$, we choose to transmit with power p_1 to make sure that the energy consumption is feasible. Then, at time $t = s_{i_1}$, the total number of bits departed is $g(p_1)s_{i_1}$, and the remaining number of bits is $B_0 - g(p_1)s_{i_1}$. Subsequently, with initial number of bits $B_0 - g(p_1)s_{i_1}$, we start from s_{i_1}, and get another lower bound on the overall transmission duration T_2, and repeat the procedure above. Through this procedure, we obtain $p_2, p_3, \ldots, p_N$, and corresponding $i_2, i_3, \ldots, i_N$, until we finish transmitting all of the bits. This procedure yields an optimum transmission strategy as proved in the following theorem.

Theorem 2 *The allocation procedure described above gives the optimal transmission policy.*

Proof: Let T be the final transmission duration given by the allocation procedure. Then, we have $B(T) = B_0$. In order to prove that the allocation is optimal, we need to show that the final transmission policy has the structure given in Theorem 1. We first prove that p_1 satisfies (14). Then, we can similarly prove that p_2, p_3, ... satisfy (14).

We know that if $T = T_1$, then it is the minimum possible transmission completion time. We know that this transmit policy will satisfy the structural properties in Theorem 1. Otherwise, the final optimal transmission time T is greater than T_1, and more harvested energy may be utilized to transmit the remaining bits. From the allocation procedure, we have

$$p_1 \leq \frac{\sum_{j=0}^{i-1} E_j}{s_i}, \quad \forall i < \tilde{i}_1 \tag{22}$$

In order to prove that p_1 satisfies (14), we need to show that

$$p_1 \leq \frac{\sum_{j=0}^{i-1} E_j}{s_i}, \quad \forall i : s_{\tilde{i}_1} \leq s_i \leq T \tag{23}$$

If we keep transmitting with power p_1, then at $T_1' = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1}$, the total number of bits departed will be

$$g(p_1)T_1' \geq g\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1}\right) T_1 = B_0 \tag{24}$$

where the inequality follows from the assumption that $g(p)/p$ decreases in p. Then, (22) guarantees that this is a feasible policy. Thus, under the optimal policy, the transmission duration T will be upper bounded by T_1', i.e.,

$$T \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1} \tag{25}$$

which implies

$$p_1 \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T} \tag{26}$$

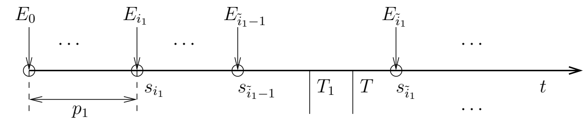

Fig. 7. Case I: $T \leq s_{\tilde{i}_1}$.

If $T \leq s_{\tilde{i}_1}$, as shown in Fig. 7, no future harvested energy is utilized for the transmissions. Then, (26) guarantees that (23) is satisfied.

If $T > s_{\tilde{i}_1}$, as shown in Fig. 8, additional energy harvested after $s_{\tilde{i}_1}$ should be utilized to transmit the data. We next prove that (23) still holds through contradiction. Assume that there exists i' with $s_{\tilde{i}_1} \leq s_{i'} \leq T$, such that (23) is not satisfied, i.e.,

$$p_1 > \frac{\sum_{j=0}^{i'-1} E_j}{s_{i'}} \triangleq p' \tag{27}$$

Then,

$$\frac{\sum_{j=0}^{i'-1} E_j}{p_1} < s_{i'} \tag{28}$$

Combining this with (25), we have $T < s_{i'}$, which contradicts with the assumption that $s_{i'} \leq T$. Thus, (23) holds, p_1 satisfies the requirement of the optimal structure in (22).

We can then prove using a similar argument that p_2, p_3 also satisfy the properties of the optimal solution. Since with fixed T, the policy with the optimal structure is unique, this procedure gives us the unique optimal transmission policy. ■

III. Scenario II: Random Bit Arrivals

In this section, we consider the situation where bits arrive during the transmissions. We assume that there is an amount of E_0 energy available at time $t = 0$, and at times s_1, s_2, ..., s_K, energy is harvested in amounts E_1, E_2, ..., E_K, respectively. We also assume that at $t = 0$, we have B_0 bits available, and at times t_1, t_2, ..., t_M, bits arrive in amounts B_1, B_2, ..., B_M, respectively. The system model is shown in Fig. 1. Our objective is again to minimize the transmission completion time, which is defined as the time that the last bit is delivered to the destination.

Denote the sequence of transmission powers as p_1, p_2, ..., p_N, and the corresponding transmission durations as l_1, l_2, ..., l_N. Then, the optimization problem becomes

$$\begin{aligned} \min_{\mathbf{p},\mathbf{l}} \quad & T \\ \text{s.t.} \quad & E(t) \leq \sum_{i:s_i<t} E_i, \quad 0 \leq t \leq T \\ & B(t) \leq \sum_{i:t_i<t} B_i, \quad 0 \leq t \leq T \\ & B(T) = \sum_{i=0}^{M} B_i \end{aligned} \tag{29}$$

where $E(\cdot)$, $B(\cdot)$ are defined in (1) and (2). We again determine the properties of the optimal transmission policy in the following three lemmas.

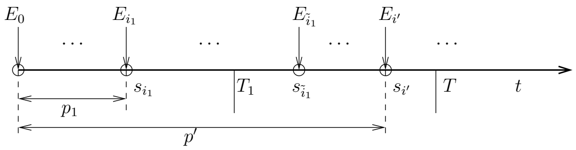

Fig. 8. Case II: $T > s_{\tilde{i}_1}$.

Lemma 4 *Under the optimal policy, the transmission rates increase monotonically, i.e.,* $r_1 \le r_2 \le \cdots \le r_N$.

Lemma 5 *The transmission power/rate remain constant between two event epoches, i.e., the rate only potentially changes when new energy is harvested or new bits arrive.*

Lemma 6 *If the transmission rate changes at an energy harvesting epoch, then the energy consumed up to that epoch equals the energy harvested up to that epoch; if the transmission rate changes at a bit arrival epoch, the number of bits departed up to that epoch equals the number of bits arrived up to that epoch.*

Lemmas 4, 5 and 6 can be proved using methods similar to those used in the proofs of Lemmas 1, 2, and 3.

We develop a similar procedure to find the optimal transmission policy. The basic idea is to keep the transmit power/rate as constant as possible throughout the entire transmission duration. Because of the additional casuality constraints due to data arrivals, we need to consider both the average data arrival rate as well as the average power the system can support for feasibility.

If $s_K \le t_M$, i.e., bits have arrived after the last energy harvest, then, all of the harvested energy will be used. First, we assume that we can use these energies to maintain a constant rate, and the transmission duration will be the solution of

$$g\left(\frac{\sum_{j=0}^{K} E_j}{T}\right) T = \sum_{j=0}^{M} B_j \tag{30}$$

Then, we check whether this constant power/rate is feasible. We check the availability of the energy, as well as the available number of bits. Let

$$i_{1e} = \arg\min_{1\le i\le K} \left\{\frac{\sum_{j=0}^{i-1} E_j}{s_i}\right\}, \quad p_1 = \frac{\sum_{j=0}^{i_{1e}-1} E_j}{s_i} \tag{31}$$

$$i_{1b} = \arg\min_{1\le i\le M} \left\{\frac{\sum_{j=0}^{i-1} B_j}{t_i}\right\}, \quad r_1 = \frac{\sum_{j=0}^{i_{1b}-1} B_j}{t_i} \tag{32}$$

We compare $\min(p_1, g^{-1}(r_1))$ with $\frac{\sum_{j=0}^{K} E_j}{T}$. If the former is greater than the latter, then the constant transmit power $\frac{\sum_{j=0}^{K} E_j}{T}$ is feasible. Thus, we achieve the minimum possible transmission completion time T. Otherwise, the constant power transmission is not feasible. We pick the transmit power to be the smaller of p_1 and $g^{-1}(r_1)$, and the duration to be the one associated with the smaller transmit power. We repeat this procedure until all the bits are transmitted.

If $s_K > t_M$, then, as in the first scenario where bits have arrived and are ready before the transmission starts, some of the harvested energy may not be utilized to transmit the bits. In this case also, we need to get a lower bound for the final transmission completion time. Let s_n be the energy harvesting epoch right after t_M. Then, starting from s_n, we compute the energy required to transmit $\sum_{j=0}^{M} B_j$ bits at a constant rate by s_i, $s_n \le s_i \le s_K$, and compare them with the total energy harvested up to that epoch, i.e., $\sum_{j=0}^{i-1} E_j$. We identify the smallest i such that the required energy is smaller than the total harvested energy, and denote it by $\tilde{i}_1$. If no such $\tilde{i}_1$ exists, we let $\tilde{i}_1 = K + 1$.

Now, we assume that we can use $\sum_{j=0}^{\tilde{i}_1-1} E_j$ to transmit $\sum_{j=0}^{M} B_j$ bits at a constant rate. We allocate the energy evenly to these bits, and the overall transmission time T_1 is the solution of

$$g\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T}\right) T = \sum_{j=0}^{M} B_j \tag{33}$$

and the corresponding constant transmit power is

$$p_1 = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1} \tag{34}$$

Next, we compare p_1 with $\frac{\sum_{j=0}^{i-1} E_j}{s_i}$ for every $i < \tilde{i}_1$, $g^{-1}(\frac{\sum_{j=0}^{i-1} B_j}{t_i})$ for $1 \le i \le M$, and $g^{-1}(\frac{\sum_{j=0}^{M} B_j}{T_1})$. If p_1 is smaller than all of these terms, then, maintaining p_1 is feasible from both energy and data points of view. The optimal policy is to keep a constant transmission rate at $g(p_1)$ with duration T_1, which is the smallest possible transmission completion time, $i_1 = \tilde{i}_1$. Otherwise, maintaining p_1 is not feasible under the given energy and data arrival realizations. This infeasibility is due to causality constraints on either the energy or the traffic, or both. Next, we identify the tightest constraint, and update the transmit power to be the power associated with that constraint. We repeat this procedure until all of the bits are delivered. The optimality of this scheme can be proved similar to the previous case.

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