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Optimal transmission schemes for parallel and fading Gaussian broadcast channels with an energy harvesting rechargeable transmitter ☆

Omur Ozel [a], Jing Yang [b], Sennur Ulukus [a,*]

[a] Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA
[b] Department of Electrical and Computer Engineering, University of Wisconsin-Madison, WI 53706, USA

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ABSTRACT

We consider an energy harvesting transmitter sending messages to two users over parallel and fading Gaussian broadcast channels. Energy required for communication arrives (is harvested) at the transmitter and a finite-capacity battery stores it before being consumed for transmission. Under off-line knowledge of energy arrival and channel fading variations, we obtain the trade-off between the performances of the users by characterizing the maximum departure region in a given interval. We first analyze the transmission with an energy harvesting transmitter over parallel broadcast channels. We show that the optimal total transmit power policy that achieves the boundary of the maximum departure region is the same as the optimal policy for the non-fading broadcast channel, which does not depend on the priorities of the users, and therefore is the same as the optimal policy for the non-fading scalar single-user channel. The optimal total transmit power can be found by a directional water-filling algorithm. The optimal splitting of the power among the parallel channels is performed in each epoch separately. Next, we consider fading broadcast channels and obtain the transmission policies that achieve the boundary of the maximum departure region. The optimal total transmit power allocation policy is found using a specific directional water-filling algorithm for fading broadcast channels. The optimal power allocation depends on the priorities of the users unlike in the case of parallel broadcast channels. Finally, we provide numerical illustrations of the optimal policies and maximum departure regions for both parallel and fading broadcast channels.

1. Introduction

A distinct characteristic of energy harvesting communication systems is that the energy required for communication arrives during the session in which communication takes place. The transmitter is able to harvest energy from nature in order to recharge its battery. The energy is modeled to arrive (be harvested) at arbitrary instants and in arbitrary amounts. Therefore, transmission schemes must adapt to the incoming energy. In this paper, we consider communication with an energy harvesting transmitter over parallel and fading AWGN broadcast channels.

In particular, we consider an energy harvesting transmitter that sends data to two receivers over parallel and fading broadcast channels as in Figs. 1 and 2. Data for the two receivers are backlogged at the transmitter buffers while arriving energy is stored in a finite-capacity battery. Service is provided to the data buffers with the cost of energy depletion from the energy buffer, i.e., the battery. Energy arrivals and channel variations are known by the transmitter a priori. Data to be sent to the receivers are assumed to be available at the data buffers before the transmission starts. Although power allocation problem in traditional systems with non-rechargeable batteries subject to average power constraints in parallel and fading broadcast channels are solved using identical techniques, off-line scheduling with rechargeable batteries in these two channel models are considerably different. We first address parallel broadcast channels. The time sequence of the power allocation and the splitting to two users are simultaneously determined for each parallel channel. Next, we consider fading broadcast channels. As the fading levels and strength order of the users vary throughout the communication, the power allocation is determined according to the joint fading variations of the users. In both scenarios, the transmitter has to adapt its transmission power with respect to the available energy and also avoid possible energy overflows due to the finite-capacity battery.

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* Corresponding author. Tel.: +1 (301) 405 4909.
E-mail addresses: omur@umd.edu (O. Ozel), yangjing@ece.wisc.edu (J. Yang), ulukus@umd.edu (S. Ulukus).

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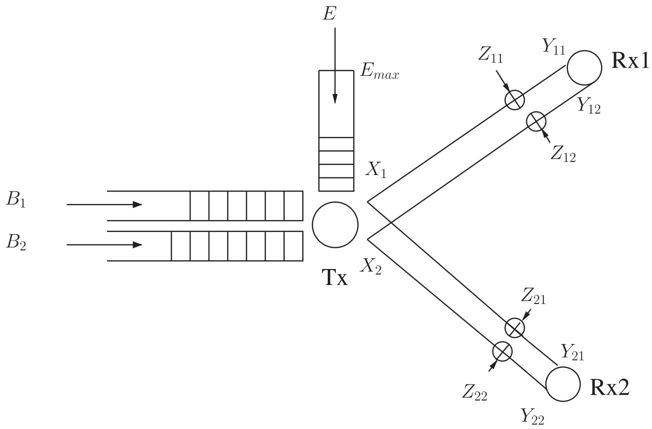

Fig. 1. The two-user parallel broadcast channel with an energy harvesting transmitter.

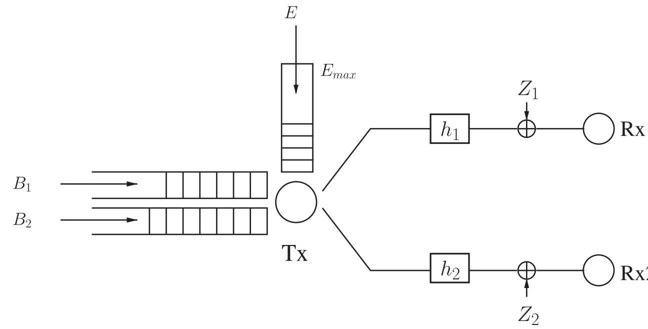

Fig. 2. The two-user fading broadcast channel with an energy harvesting transmitter.

There has been recent research effort on understanding data transmission with an energy harvesting transmitter that has a rechargeable battery [1–7]. In [1], data transmission with energy harvesting sensors is considered, and optimal on-line policy for controlling admissions into the data buffer is derived using a dynamic programming framework. In [2], energy management policies which stabilize the data queue are proposed for single-user communication and some delay optimality properties are derived under a linearity assumption for the power-rate relation. In [3], throughput optimal energy allocation is studied for energy harvesting systems in a time constrained slotted setting. In [4,5], minimization of the transmission completion time is considered in an energy harvesting single-user system and the optimal solution is obtained analytically and using a geometric algorithm. In [6], energy harvesting transmitters with batteries of finite energy storage capacity are considered and the problem of throughput maximization by a deadline is solved in a static channel. In [7], optimal off-line and on-line transmission policies are given for a single-user energy harvesting transmitter operating in a fading channel. In [8,9], optimal off-line policies are developed for the static AWGN broadcast channel with an infinite capacity battery, concurrently and independently. References [10,11] extend the broadcasting framework to the case of a finite-capacity battery energy harvesting transmitter.

In a previous related line of research, transmission scheduling for maximum rate or minimum energy under delay constraints has been considered in [12–17]. In [12], optimal off-line packet scheduling for energy minimization is solved in a delay constrained single-user data link and in [13], the framework is extended for multiple access, broadcast and fading data links. In [14], the energy minimal transmission for a single-user data link is solved using a calculus approach that incorporates various quality of service constraints. In [15], optimal energy allocation to a fixed number of time slots is derived under time-varying channel gains and with off-line and on-line knowledge of the channel state at the transmitter. In [16], delay constrained capacity of fading channels is found under causal feedback using dynamic programming. In [17], capacity of a two-user fading broadcast channel is determined under stringent delay constraints.

In this paper, we extend the line of research in the off-line optimal scheduling in energy harvesting communication systems for the parallel and fading broadcast channels. As the users utilize the common resources, which are the harvested energy and the wireless communication medium, there is a trade-off between the performances of the users. We characterize this trade-off by obtaining the maximum departure region [8,11] by a deadline T and determine the optimal off-line policies that achieve the boundary of the maximum departure region. We first investigate recently developed results for the non-fading broadcast channel in [10,11]. Next, we consider off-line scheduling for energy harvesting transmitters over parallel broadcast channels. We show that the optimal total transmit power policy that achieves the boundary of the maximum departure region is the same as the optimal policy for the non-fading scalar broadcast channel, which does not depend on the priorities of the users, and therefore is the same as the optimal policy for the non-fading scalar single-user channel. The optimal policy is found by a directional water-filling algorithm which is based on transferring energies from past to the future. The amount of energy transfer is limited by the finite battery capacity constraint. The power is split to each parallel channel separately in each epoch. We then consider off-line scheduling for energy harvesting transmitters over fading broadcast channels. We show that in the optimal policy that achieves the boundary of the maximum departure region, energy allocation in each epoch is determined by a directional water-filling algorithm [7] that is specific to the fading broadcast channel. In particular, water level in between two energy arrivals is calculated by using the water-filling scheme described in [18] or the greedy power allocation in [19]. If the water level is higher on the right, no energy is transferred; otherwise some energy is transferred to the future. Unlike the case of parallel broadcast channels, in the case of fading broadcast channels, the total transmit power policies achieving different points on the boundary of the maximum departure region depend on the priorities of the users. Finally, we numerically examine the resulting maximum departure regions for parallel and fading broadcast channels in a deterministic setting.

2. The channel and energy models

In this paper, we consider two different channel models, namely parallel broadcast channels and fading broadcast channels. Although the treatment of these two channel models in traditional systems with non-rechargeable batteries subject to average power constraints are equivalent [18,19], the extra dimension created due to the battery energy variations at the transmitter leads to significant differences between these two channel models in the context of off-line broadcast scheduling. In the following, we provide the details of these two channel models as well as the energy model.

2.1. The parallel broadcast channel model

In a two-user parallel broadcast channel, one transmitter sends data to two receivers over independent parallel channels. The model is depicted in Fig. 1. We consider the case where there are two parallel channels only. The generalization to more than two parallel channels is straightforward, and left out for brevity and clarity of presentation in this paper.

The received signals at the two receivers are

$$Y_{1i} = X_i + Z_{1i}, \quad i = 1,2 \tag{1}$$

$$Y_{2i} = X_i + Z_{2i}, \quad i = 1,2 \tag{2}$$

where X_i is the signal transmitted in the ith parallel channel, and Z_{1i} and Z_{2i} are Gaussian noises with variances σ_{1i}^2 and σ_{2i}^2, respectively. If $\sigma_{1i}^2 \leqslant \sigma_{2i}^2$ for all i, or $\sigma_{2i}^2 \leqslant \sigma_{1i}^2$ for all i, then the overall channel is degraded in favor of user 1 or user 2, respectively, and hence the problem reduces to the scheduling problem over a scalar non-fading broadcast channel as in [8–11]. Therefore, we consider the case $\sigma_{11}^2 < \sigma_{21}^2$ and $\sigma_{12}^2 > \sigma_{22}^2$ where the overall broadcast channel is not degraded.

Assuming that the transmitter transmits with power P, the achievable rate region for this two-user parallel broadcast channel is [19,20]

$$R_1 \leqslant \frac{1}{2}\log_2\left(1 + \frac{\alpha_1 \beta P}{\sigma_{11}^2}\right) + \frac{1}{2}\log_2\left(1 + \frac{\alpha_2(1-\beta)P}{(1-\alpha_2)(1-\beta)P + \sigma_{12}^2}\right) \tag{3}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha_2)(1-\beta)P}{\sigma_{22}^2}\right) + \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha_1)\beta P}{\alpha_1 \beta P + \sigma_{21}^2}\right) \tag{4}$$

where βP is the power allocated to the first parallel channel, and $(1-\beta)P$ is the power allocated to the second parallel channel, α_1 and α_2 are the fractions of powers spent for the message transmitted to user 1 in each parallel channel. Note that even though the overall channel is not degraded, there is no constraint on the sum rate in the expressions that define the capacity region in (3) and (4) since individual channels are degraded. By varying $\alpha_1 \in [0,1],\ \alpha_2 \in [0,1]$ and $\beta \in [0,1]$, we obtain a family of achievable regions and their union is the capacity region. Any operating point on the boundary of the capacity region is fully characterized by solving for the power allocation policy that maximizes $\mu_1 R_1 + \mu_2 R_2$ for some (μ_1, μ_2). For any μ_1, μ_2, there exist $P^*, \alpha_1^*, \alpha_2^*$ and β^* that achieve the corresponding point on the boundary of the capacity region [18,19].

2.2. The fading broadcast channel model

The fading broadcast channel model is depicted in Fig. 2. The received signals at the two receivers are

$$Y_1 = \sqrt{h_1}X + Z_1 \tag{5}$$

$$Y_2 = \sqrt{h_2}X + Z_2 \tag{6}$$

where X is the transmit signal, Z_1, Z_2 are Gaussian noises with zero-mean and variances σ_1^2 and σ_2^2, respectively, and h_1, h_2 are the (squared) fading coefficients[1] for receivers 1 and 2, respectively. As in [18], we combine the effects of fading and noise power, and obtain an equivalent broadcast channel by letting $n_1 = \frac{\sigma_1^2}{h_1}$ and $n_2 = \frac{\sigma_2^2}{h_2}$. If the channel fade levels are constant at h_1, h_2, and the transmitter transmits with power P, the resulting broadcast channel capacity region is [20]:

$$R_1 \leqslant \frac{1}{2}\log_2\left(1 + \frac{\alpha P}{(1-\alpha)P\mathbf{1}(n_1 > n_2) + n_1}\right) \tag{7}$$

$$R_2 \leqslant \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)P}{\alpha P\mathbf{1}(n_2 > n_1) + n_2}\right) \tag{8}$$

where α is the fraction of the power spent for the message transmitted to user 1, and $\mathbf{1}(x > y)$ is the indicator function for the event $x > y$. We call the receiver which observes smaller combined noise power the stronger receiver and the other one the weaker receiver. That is, receiver 1 is the stronger user if $n_1 < n_2$ and receiver 2 is the stronger user if $n_2 < n_1$. Note that changes in the fading levels of the channels during the communication session causes time variation in the strength order of the receivers.

2.3. Energy and power-rate models

In two-user energy harvesting parallel and fading broadcast channels, the transmitter has three queues as in Figs. 1 and 2: two data queues where data packets for the two receivers are stored, and an energy queue where the arriving (harvested) energy is stored. The energy queue, i.e., the battery, can store at most E_{max} units of energy, which is used for transmission only, i.e., energy required for processing is not considered.

We consider an off-line setting where the changes that occur in the energy levels throughout the communication session are known by the transmitter a priori. In the fading broadcast channel, the changes in the fade levels are also known by the transmitter a priori. Performance of any transmission policy with a priori knowledge provides an upper bound for that of a real time system. In the fading broadcast channel, the fading and energy levels change at discrete time instants $t_1^f, t_2^f, \ldots, t_n^f, \ldots$ and $t_1^e, t_2^e, \ldots, t_n^e, \ldots$, respectively, as shown in Fig. 3. Note that a change in the fading level means any change in the joint fading state (h_1, h_2). We define an epoch as a time interval in which no energy arrival or channel fade level change occurs as shown in Fig. 3. An epoch in the parallel broadcast channels scenario is the time interval between two energy harvests as the channel gains do not vary. In the fading broadcast channel, we extend the definition of energy arrival sequence for the time instants at which a fading change occurs. In particular, the input energy for epoch i is denoted as E_{i-1} and it is equal to the amount of incoming energy if the epoch starts with an energy arrival; if epoch i starts with a variation in the fading level without an energy arrival, $E_{i-1} = 0$. Finally, we let ℓ_i denote the length of the ith epoch.

Whenever an input signal x is transmitted with power p in an epoch of duration ℓ in which the channel fades are constant at the levels h_1 and $h_2, R_1\ell$ and $R_2\ell$ bits of data are served out from the backlogs of receivers 1 and 2 at the transmitter, with the cost of $p\ell$ units of energy depletion from the energy queue. Here, (R_1, R_2) is the rate allocation for this epoch. (R_1, R_2) must reside in the corresponding capacity region. In particular, for the parallel channels scenario, (R_1, R_2) must satisfy (3) and (4), and in the fading broadcast channel scenario, (R_1, R_2) must reside in the capacity region of the two-user AWGN broadcast channel $\mathcal{C}_{n_1,n_2}(P)$, indexed by the noise variances n_1 and n_2, which vary during the communication session. Extending this for continuous time, if at time t the transmit power is $P(t)$ and the noise variances are $n_1(t) = \sigma_1^2/h_1(t)$ and $n_2(t) = \sigma_2^2/h_2(t)$, the instantaneous rate pairs $(R_1(t), R_2(t))$ reside in the corresponding capacity region, i.e., $(R_1(t), R_2(t)) \in \mathcal{C}_{n_1(t),n_2(t)}(P(t))$.

The transmission policy in the parallel broadcast channel is comprised of $P(t)$, the total power, $\beta(t) \in [0,1]$, the power share of the 1st parallel channel, and $\alpha_1(t) \in [0,1]$ and $\alpha_2(t) \in [0,1]$, the power shares of user 1 in the 1st and 2nd parallel channels, respectively. In fading broadcast channels, transmission policy is comprised of the total power $P(t)$ and the portion of the total transmit power $\alpha(t) \in [0,1]$ that is allocated for user 1. Therefore, in parallel and fading broadcast channels, the total energy consumed by the transmitter up to time t can be expressed as $\int_0^t P(\tau)d\tau$. Due to the finiteness of the battery capacity, at any time t, if the unconsumed energy is greater than E_{max}, only E_{max} can be stored in the battery and the rest of the energy is wasted due to energy overflow. This may happen only at the instants of energy arrivals. Therefore, the total removed energy from the battery at

[1] We note that the model can be generalized to a broadcast channel with conventional complex baseband fading coefficients after proper scalings that are inconsequential for our analysis.

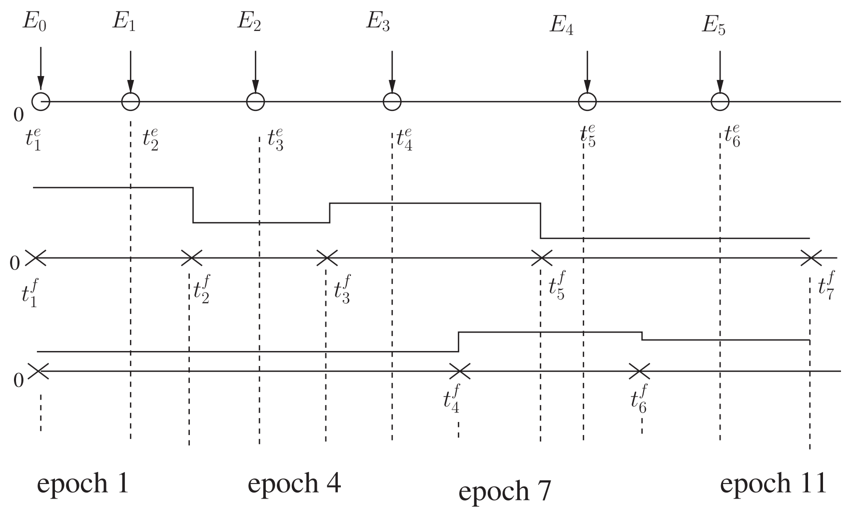

Fig. 3. The energy arrivals, channel variations and epochs.

$s_k, E_r(s_k)$, including the consumed part and the wasted part, can be expressed recursively as

$$E_r(s_k^+) = \max\left\{E_r(s_{k-1}^+) + \int_{s_{k-1}}^{s_k} P(\tau)d\tau, \left(\sum_{j=0}^{k} E_j - E_{max}\right)^+\right\},$$
$$k = 1, 2, \ldots \qquad (9)$$

where $(x)^+ = \max\{0, x\}$, and s_k^+ should be interpreted as $s_k + \epsilon$ for arbitrarily small $\epsilon > 0$. In addition, $E_r(s_0) = 0$. We can extend the definition of E_r for the times $t \neq s_k$ as:

$$E_r(t) = E_r(s_{d_+(t)}^+) + \int_{s_{d_+(t)}}^{t} P(\tau)d\tau \qquad (10)$$

where $d_+(t) = \max\{i : s_i \leqslant t\}$. As the transmitter cannot utilize the energy that has not arrived yet, the transmission policy is subject to an energy causality constraint. The removed energy $E_r(t)$ cannot exceed the total energy arrival during the communication. This constraint is mathematically stated as follows:

$$E_r(t) \leqslant \sum_{i=0}^{d_-(t)} E_i, \quad \forall t \in [0, T] \qquad (11)$$

where $d_-(t) = \max\{i : s_i < t\}$. As the energies arrive at discrete times, the causality constraint reduces to inequalities that have to be satisfied at the times of energy arrivals:

$$E_r(s_{k-1}^+) + \int_{s_{k-1}}^{s_k} P(\tau)d\tau \leqslant \sum_{i=0}^{k-1} E_i, \quad \forall k \qquad (12)$$

An illustration of the energy removal curve $E_r(t)$ and the causality constraint is shown in Fig. 4. The upper curve in Fig. 4 is the total energy arrival curve and the lower curve is obtained by subtracting E_{max} from the upper curve. The causality constraint imposes $E_r(t)$ to remain below the upper curve. Moreover, $E_r(t)$ always remains above the lower curve due to (9) and (10). In Fig. 4, the energy in the battery exceeds E_{max} at the time of the third energy arrival and consequently, some energy is removed from the battery without being utilized for data transmission. Energy removal from the battery continues due to only the data transmission in (s_3, s_4) interval and hence the energy removal curve approaches the total energy arrival curve indicating that the battery energy is decreasing. In general, battery energy is non-increasing in (s_{i-1}, s_i) interval for all i.

As observed in Fig. 4, energy is wasted due to energy overflow if $E_r(t)$ intersects the lower curve at the vertically rising parts at the energy arrival instants. Therefore, a transmission policy guarantees no-energy-overflow if the following constraint is satisfied:

$$\int_0^t P(\tau)d\tau \geqslant \left(\sum_{i=0}^{d_+(t)} E_i - E_{max}\right)^+, \quad \forall t \in [0, T] \qquad (13)$$

The constraint in (13) imposes that at least $\sum_{i=0}^{k} E_i - E_{max}$ amount of energy has been consumed (including both the data transmission and the energy overflow) by the time the kth energy arrives so that the battery can accommodate E_k at time s_k. If a policy satisfies (13), the max in (9) always yields the first term in it. Therefore, the causality constraint in (12) reduces to the following:

$$\int_0^t P(\tau)d\tau \leqslant \sum_{i=0}^{d_-(t)} E_i, \quad \forall k \qquad (14)$$

This is shown in Fig. 5 in which the total energy curve of the policy does not intersect the lower curve at the vertically rising parts (at the energy arrival instants) so that no energy is wasted due to energy overflows. Hence, the causality constraint simplifies to the condition that the total energy curve must lie below the upper curve in Fig. 5.

3. The maximum departure region

In both parallel and fading broadcast channels, the performances of user 1 and user 2 are strongly coupled as they are yielded by the utilization of the common resources, which are the harvested energy and the shared wireless communication channel. In this section, we characterize the trade-off between the performances of user 1 and user 2 by finding the region of bits sent for receivers 1 and 2 in the interval $[0, T]$ with off-line knowledge of energy and fading variations. The number of bits sent for users 1 and 2 are:

$$B_1 = \int_0^T R_1(\tau)d\tau \qquad (15)$$

$$B_2 = \int_0^T R_2(\tau)d\tau \qquad (16)$$

The instantaneous rates $R_1(t)$ and $R_2(t)$ are determined as a function of the instantaneous power policy $P(t)$ as described in power-rate model in Section 2.3. Next, we define the maximum departure region characterizing the bits sent for the users.

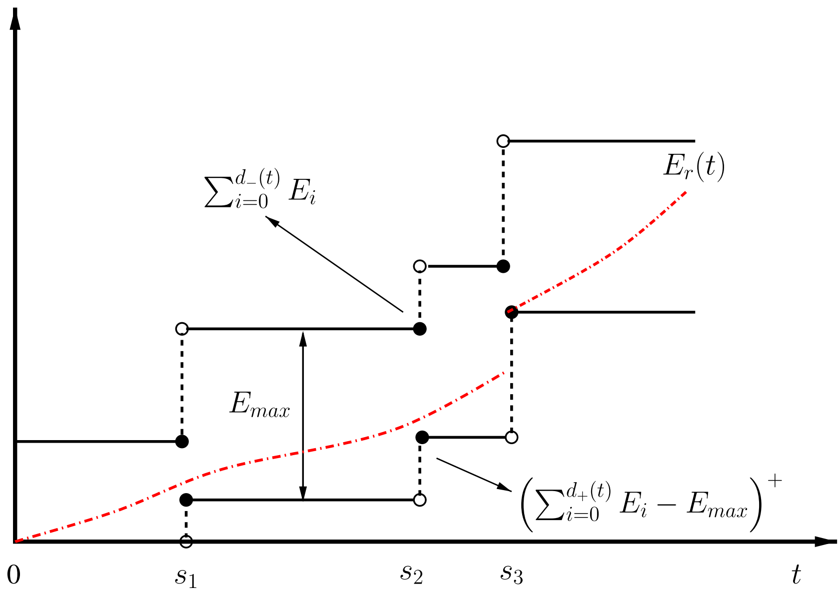

Fig. 4. The total removed energy curve $E_r(t)$. The jump at s_3 represents an energy overflow because of the finite-capacity battery.

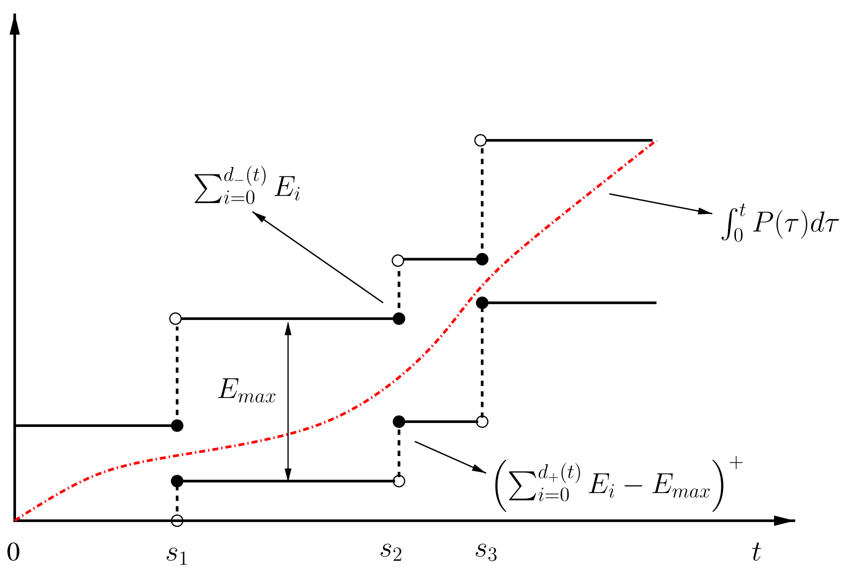

Fig. 5. Energy causality constraint and no-energy-overflow constraint are depicted as cumulative energy curves and the power consumption curve of a transmission policy that simultaneously satisfies these two constraints by lying in between these two curves.

Definition 1. For any fixed transmission duration T, the maximum departure region, denoted as $\mathcal{D}(T)$, is the union of $\mathcal{R}(B_1, B_2) = \{(b_1, b_2) : 0 \leqslant b_1 \leqslant B_1,\ 0 \leqslant b_2 \leqslant B_2\}$ where (B_1, B_2) is the total number of bits sent by some power allocation policy that satisfies energy causality and no-energy-overflow conditions over the duration $[0, T)$.

We have the following lemma, the proof of which can be carried out following the proofs of Lemma 2 in [8] and Lemma 1 in [11] and hence is skipped here for brevity.

Lemma 1. *For both parallel and fading broadcast channels, $\mathcal{D}(T)$ is a convex region.*

We note that a transmission policy that violates the no-energy-overflow condition is always strictly inside $\mathcal{D}(T)$; therefore, without losing optimality we restrict the feasible set to the policies that allow no energy overflows. In the following analysis, we call any policy that satisfies energy causality and no-energy-overflow conditions *feasible*. We call a feasible policy *optimal* if it achieves the boundary of $\mathcal{D}(T)$.

3.1. $\mathcal{D}(T)$ for parallel broadcast channels

In parallel broadcast channels, the instantaneous rates $r_1(t)$ and $r_2(t)$ allocated for users 1 and 2 are determined as a function of the instantaneous power, $P(t)$, power share of the 1st channel, $\beta(t)$, and the power shares of user 1 in the ith channel, $\alpha_i(t),\ i = 1, 2$, via (3) and (4). The instantaneous power, $P(t)$, is subject to the energy causality and no-energy-overflow conditions as in (14) and (13), respectively. We let N denote the number of energy arrivals in the $[0, T]$ interval.

Due to the convexity of $\mathcal{D}(T)$ in Lemma 1 and the convex power-rate relation, an optimal policy should remain constant in any epoch (c.f. Lemma 1 in [8] and Lemma 2 in [4,5]). Therefore, we consider a power policy as a sequence of powers allocated for each epoch $\{p_i\}_{i=1}^{N+1}$ with the 1st channel's share $\{\beta_i\}_{i=1}^{N+1}$, the power share of user 1 in each channel $\{(\alpha_{i1}, \alpha_{i2})\}_{i=1}^{N+1}$. Then, the energy causality and no-energy-overflow conditions in (14) and (13) reduce to the following constraints, respectively, which are described by a finite sequence of powers:

$$\sum_{i=1}^{k} p_i \ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad k = 1, \ldots, N+1 \tag{17}$$

$$\sum_{i=1}^{k} p_i \ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max} \right)^+, \quad k = 1, \ldots, N \tag{18}$$

Here (17) is due to the energy causality constraint in (14) and (18) is due to the no-energy-overflow condition in (13). We define the following functions:

$$r_1(\alpha_1, \alpha_2, \beta, p) = \frac{1}{2}\log_2\left(1 + \frac{\alpha_1 \beta p}{\sigma_{11}^2}\right) + \frac{1}{2}\log_2\left(1 + \frac{\alpha_2 (1-\beta) p}{(1-\alpha_2)(1-\beta)p + \sigma_{12}^2}\right) \tag{19}$$

$$r_2(\alpha_1, \alpha_2, \beta, p) = \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha_1)\beta p}{\alpha_1 \beta p + \sigma_{21}^2}\right) + \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha_2)(1-\beta)p}{\sigma_{22}^2}\right) \tag{20}$$

which are the rates achieved by users 1 and 2, respectively, if p is allocated to the channels with the first parallel channel's share βp, and user 1's power share (α_1, α_2) in each channel. By Lemma 1, any point on the boundary of the maximum departure region $\mathcal{D}(T)$ can be characterized by solving the following optimization problem:

$$\begin{aligned}
\max_{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\beta}, \mathbf{p}} \quad & \mu_1 \sum_{i=1}^{N+1} r_1(\alpha_{1i}, \alpha_{2i}, \beta_i, p_i)\ell_i + \mu_2 \sum_{i=1}^{N+1} r_2(\alpha_{1i}, \alpha_{2i}, \beta_i, p_i)\ell_i \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_i \ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad \forall k \\
& \sum_{i=1}^{k} p_i \ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max} \right)^+, \quad \forall k \\
& 0 \leqslant \alpha_{ik} \leqslant 1, \quad 0 \leqslant \beta_i \leqslant 1, \quad p_k \geqslant 0, \quad i = 1, 2, \quad \forall k
\end{aligned} \tag{21}$$

In (21), $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\beta}, \mathbf{p}$ collectively denote the vector of total powers and power shares for the parallel channels and users. The optimization problem (21) is not a convex problem as the variables $p_i, \beta_i, \alpha_{1i}$ and α_{2i} appear in product forms in the expression of $r_i(\alpha_1, \alpha_2, \beta, p)$, causing it to be non-concave in α_i, p and β jointly. However, for any given $\alpha_1, \alpha_2, \beta$ we note that $\mu_1 r_1(\alpha_1, \alpha_2, \beta, p) + \mu_2 r_2(\alpha_1, \alpha_2, \beta, p)$ is concave with respect to p. Using this property, we solve (21) in two steps. We optimize over $\alpha_{1i}, \alpha_{2i}, \beta_i$ first and then over the total power p_i. The details of the optimal policy are presented in Section 5.

3.2. $\mathcal{D}(T)$ for fading broadcast channels

In fading broadcast channels, the instantaneous rates $r_1(t)$ and $r_2(t)$ allocated for users 1 and 2 are determined as a function of the instantaneous power, $P(t)$ and the power share of user 1, $\alpha(t)$, via (7) and (8). The instantaneous power, $P(t)$, is subject to the energy causality and no-energy-overflow conditions as in (14) and (13), respectively. We let N denote the number of energy arrivals and K denote the number of changes in the joint fading level in the $[0, T]$ interval. We assume that fading variations and energy arrivals occur at distinct time instants so that the number of epochs in $[0, T]$ interval is $N+K+1$. If an energy arrival and a fading variation occur at the same instant, the number of epochs is less than $N+K+1$.

Due to the convexity of $\mathcal{D}(T)$ in Lemma 1 and the convex power-rate relation, an optimal policy should remain constant in any epoch (c.f. Lemma 1 in [8] and Lemma 2 in [4,5]). Therefore, the policy is a sequence of powers $\{p_i\}_{i=1}^{N+K+1}$ and user 1's power share $\{\alpha_i\}_{i=1}^{N+K+1}$. The sequence of noise variances of the equivalent broadcast channels is $\{(n_{1i}, n_{2i})\}_{i=1}^{N+K+1}$. Then, the causality and no-energy-overflow conditions in (14) and (13) reduce to the following constraints, respectively, which are described by a finite sequence of powers:

$$\sum_{i=1}^{k} p_i \ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad k = 1, \ldots, N+K+1 \tag{22}$$

$$\sum_{i=1}^{k} p_i \ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max} \right)^+, \quad k = 1, \ldots, N+K \tag{23}$$

Here (22) is due to the energy causality constraint in (14) and (23) is due to the no-energy-overflow condition in (13). We define the following functions:

$$r_1(n_1, n_2, \alpha, p) = \frac{1}{2}\log_2\left(1 + \frac{\alpha p}{(1-\alpha)p\mathbf{1}(n_1 > n_2) + n_1}\right) \tag{24}$$

$$r_2(n_1, n_2, \alpha, p) = \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)p}{\alpha p\mathbf{1}(n_2 > n_1) + n_2}\right) \tag{25}$$

which are the rates achieved by users 1 and 2, respectively, in the fading broadcast channel when power is p and power share of user 1 is α. By Lemma 1, any point on the boundary of $\mathcal{D}(T)$ can be characterized by solving the following optimization problem for some $\mu_1, \mu_2 \geqslant 0$:

$$\begin{aligned}
\max_{\mathbf{p}, \boldsymbol{\alpha}} \quad & \mu_1 \sum_{i=1}^{N+K+1} r_1(n_{1i}, n_{2i}, \alpha_i, p_i)\ell_i + \mu_2 \sum_{i=1}^{N+K+1} r_2(n_{1i}, n_{2i}, \alpha_i, p_i)\ell_i \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_i \ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad \forall k \\
& \sum_{i=1}^{k} p_i \ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max} \right)^+, \quad \forall k \\
& 0 \leqslant \alpha_k \leqslant 1, \quad p_k \geqslant 0, \quad \forall k
\end{aligned} \tag{26}$$

where $\mathbf{p}, \boldsymbol{\alpha}$ denote the vector of total powers and the power shares of user 1, respectively. The optimization problem in (26) is not a convex problem as the variables p_i, α_i appear in a product form in the expression of $r_i(n_1, n_2, \alpha, p)$, causing it to be non-concave in α_i and p_i jointly. However, $\mu_1 r_1(n_1, n_2, \alpha, p) + \mu_2 r_2(n_1, n_2, \alpha, p)$ is concave with respect to p for any given α. We will solve (26) using this property. The details of the optimal policy for the fading broadcast channel is presented in Section 6.

For ease of exposition, we first consider the optimal policy in the non-fading broadcast channel in the next section.

4. Optimal policy for non-fading broadcast channel

In this section, we review the results presented in [10] for the non-fading broadcast channel. The results were presented from a rate perspective in [10]. Here, we present them alternatively from a power perspective. We set the fading coefficients as $h_1 = 1$ and $h_2 = 1$ and in addition, we assume without loss of generality that $\sigma_1^2 = 1$ and $\sigma_2^2 = \sigma^2 > 1$ so that user 1 is the stronger user and user 2 is the weaker user. The main step is to view the maximization problem in (21) and (26) for the scalar non-fading broadcast channel as a sequence of single-variable maximization problems over the variable α_i given p_i as follows:

$$\max_{\alpha_i} \quad \frac{\mu_1}{2}\log_2(1+\alpha_i p_i) + \frac{\mu_2}{2}\log_2\left(1+\frac{(1-\alpha_i)p_i}{\alpha_i p_i+\sigma^2}\right)$$
$$\text{s.t.} \quad 0 \leqslant \alpha_i \leqslant 1 \tag{27}$$

For any given p_i, this optimization problem has a unique solution for α_i. Let us define a function $\alpha^*(p) : \mathbf{R}^+ \to [0,1]$ which denotes the solution of the problem in (27) for $p_i = p$. We obtain $\alpha^*(p)$ as follows: let us further denote $\mu = \frac{\mu_1}{\mu_2}$. If $\mu \leqslant 1$ then $\alpha^*(p) = 1$ for all p. If $\mu \geqslant \sigma^2$, then $\alpha^*(p) = 0$ for all p. For $1 < \mu < \sigma^2$, we have

$$\alpha^*(p) = \begin{cases} 1, & 0 \leqslant p \leqslant \frac{\sigma^2-\mu}{\mu-1} \\ \frac{1}{p}\frac{\sigma^2-\mu}{\mu-1}, & p \geqslant \frac{\sigma^2-\mu}{\mu-1} \end{cases} \tag{28}$$

Let us define the following function:

$$f(p) \triangleq \frac{\mu_1}{2}\log_2(1+\alpha^*(p)p) + \frac{\mu_2}{2}\log_2\left(1+\frac{(1-\alpha^*(p))p}{\alpha^*(p)p+\sigma^2}\right) \tag{29}$$

Lemma 2. ([11]) *$f(p)$ is monotone increasing and strictly concave function of p.*

Then, the optimization problem in (21) for the scalar non-fading case can be rewritten as an optimization problem only in terms of p_i as follows:

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{N+1} f(p_i)\ell_i$$
$$\text{s.t.} \quad \sum_{i=1}^{k} p_i\ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad \forall k$$
$$\sum_{i=1}^{k} p_i\ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max}\right)^+, \quad \forall k$$
$$p_k \geqslant 0, \quad \forall k \tag{30}$$

The optimization problem in (30) is a convex optimization problem. The objective function is strictly concave by Lemma 2 and the feasible set is a convex set. We write the Lagrangian function as:

$$\mathcal{L} = \sum_{i=1}^{N+1} f(p_i)\ell_i - \sum_{k=1}^{N+1}\lambda_k\left(\sum_{i=1}^{k}p_i\ell_i - \sum_{i=0}^{k-1}E_i\right) - \sum_{k=1}^{N}\eta_k\left(\left(\sum_{i=0}^{k}E_i - E_{max}\right)^+ - \sum_{i=1}^{k}p_i\ell_i\right) \tag{31}$$

Using the KKT optimality and complementary slackness conditions on $\mathcal{L}$, it can be shown [11] that the unique optimal total transmit power allocation is the same for all (μ_1,μ_2). This unique optimal total transmit power allocation can be found by the directional water-filling algorithm introduced in [7]. Alternatively, this unique optimal total power allocation can be found by using the feasible energy tunnel approach proposed in [6]. Note that the structures of the two alternative algorithms in [6,7], as well as the ones in [4,5] for the unconstrained battery case, are determined by the strict concavity of the rate-power relation. We obtained the same structure in the broadcast channel because of the strict concavity of $f(p)$ due to Lemma 2.

After finding the optimal total power allocation $p_i^*, i = 1, \ldots, N+1$, we can find the solution of the original problem in (26) by finding the optimal α_i^*, $i = 1,\ldots,N+1$, via $\alpha_i^* = \alpha^*(p_i)$ using (28). We first note that due to the degradedness of the second user, when $\frac{\mu_2}{\mu_1} \leqslant 1$, the total power p_i is allocated to the first user only and no bits are transmitted for the second user. When $1 < \frac{\mu_2}{\mu_1}$, we define the cut-off power level as

$$P_c = \left[\frac{\sigma^2-\mu}{\mu-1}\right]^+ \tag{32}$$

where $\mu = \frac{\mu_2}{\mu_1}$. A point on the boundary of $\mathcal{D}(T)$, which is equally represented by μ, is achieved by the following policy: for $1 < \mu$, if in an epoch the total transmit power level is below P_c in (32), then, only the stronger user's data is transmitted; otherwise, both users' data are transmitted and the stronger user's power share is P_c. For $\mu \leqslant 1$, only the stronger user's data is transmitted. Therefore, the optimal policies that achieve the boundary of $\mathcal{D}(T)$ have a common total power sequence and its splitting between the two users depends on μ_1, μ_2 through $\mu = \frac{\mu_2}{\mu_1}$. For different values of μ, the optimal policy achieves different boundary points on the maximum departure region. Varying the value of μ traces the boundary of $\mathcal{D}(T)$.

5. Optimal policy for parallel broadcast channels

The optimization problem in (21) can be cast as a sequence of optimization problems of the following form given the power p:

$$\max_{\alpha_1,\alpha_2,\beta} \quad \mu_1 r_1(\alpha_1,\alpha_2,\beta,p) + \mu_2 r_2(\alpha_1,\alpha_2,\beta,p)$$
$$\text{s.t.} \quad 0 \leqslant \alpha_1,\alpha_2,\beta \leqslant 1 \tag{33}$$

Note that given β and p, optimal α_1 and α_2 can be separately calculated. In particular, (33) is solved at $\alpha_1 = \alpha_1^*(\beta,p)$ and $\alpha_2 = \alpha_2^*(\beta,p)$. If $\frac{\mu_2}{\mu_1} \leqslant 1$, $\alpha_1^*(\beta,p) = 1$ while if $\frac{\mu_2}{\mu_1} \geqslant \frac{\sigma_{21}^2}{\sigma_{11}^2}$, $\alpha_1^*(\beta,p) = 0$ for all β. On the other hand, if $1 < \frac{\mu_2}{\mu_1} < \frac{\sigma_{21}^2}{\sigma_{11}^2}$, we have

$$\alpha_1^*(\beta,p) = \begin{cases} 1, & 0 \leqslant \beta p \leqslant \frac{\mu_2\sigma_{11}^2-\mu_1\sigma_{21}^2}{\mu_1-\mu_2} \\ \frac{1}{\beta p}\frac{\mu_2\sigma_{11}^2-\mu_1\sigma_{21}^2}{\mu_1-\mu_2}, & \beta p \geqslant \frac{\mu_2\sigma_{11}^2-\mu_1\sigma_{21}^2}{\mu_1-\mu_2} \end{cases} \tag{34}$$

Similarly, if $\frac{\mu_1}{\mu_2} \leqslant 1$, $\alpha_2^*(\beta,p) = 0$ while if $\frac{\mu_1}{\mu_2} \geqslant \frac{\sigma_{12}^2}{\sigma_{22}^2}$ then $\alpha_2^*(\beta,p) = 1$ for all β. If $1 < \frac{\mu_1}{\mu_2} < \frac{\sigma_{12}^2}{\sigma_{22}^2}$,

$$\alpha_2^*(\beta,p) = \begin{cases} 0, & 0 \leqslant (1-\beta)p \leqslant \frac{\mu_1\sigma_{22}^2-\mu_2\sigma_{12}^2}{\mu_2-\mu_1} \\ 1-\frac{1}{(1-\beta)p}\frac{\mu_1\sigma_{22}^2-\mu_2\sigma_{12}^2}{\mu_2-\mu_1}, & (1-\beta)p \geqslant \frac{\mu_1\sigma_{22}^2-\mu_2\sigma_{12}^2}{\mu_2-\mu_1} \end{cases} \tag{35}$$

Hence, (33) is equivalent to the following given p:

$$\max_{0\leqslant\beta\leqslant1} \mu_1 r_1^*(\beta,p) + \mu_2 r_2^*(\beta,p) \tag{36}$$

where $r_1^*(\beta,p) = r_1(\alpha_1^*(\beta,p),\alpha_2^*(\beta,p),\beta,p)$ and $r_2^*(\beta,p) = r_2(\alpha_1^*(\beta,p), \alpha_2^*(\beta,p),\beta,p)$. Note that in view of Lemma 2, the objective function in (36) is strictly concave with respect to the two power levels $p_1 = \beta p$ and $p_2 = (1-\beta)p$ allocated to the two parallel channels. This, in turn, implies that the objective function in (36) is strictly concave with respect to β. The solution of (36) has a water-filling interpretation. Working on the optimal α_1 and α_2 in (34) and (35), one can show that

$$\beta^* p = \max_{u\in\{1,2\}}\left(\mu_u\lambda - \sigma_{u1}^2\right)^+ \tag{37}$$

$$(1-\beta^*)p = \max_{u\in\{1,2\}}\left(\mu_u\lambda - \sigma_{u2}^2\right)^+ \tag{38}$$

where λ is the water level and β^* is the optimizer of (36). The water level λ is found by a greedy power allocation algorithm [18,19]. Power is incrementally allocated to the parallel channel that yields the maximum increase in the objective function in (36): for small power values, only a single parallel channel is allocated power. As the power is further increased, both parallel channels are allocated power. In the extreme cases, only single users are allocated power and the power is split over the parallel channels by single-user water-filling: If $\frac{\mu_2}{\mu_1} \leqslant \frac{\sigma_{22}^2}{\sigma_{12}^2}$, then all the power is allocated to user 1;

if $\frac{\mu_2}{\mu_1} \geqslant \frac{\sigma_{21}^2}{\sigma_{11}^2}$, then all the power is allocated for user 2. The outcome of the optimization problem depends on the power p. Let us define

$$g(p) = \max_{0\leqslant\beta\leqslant 1} \mu_1 r_1^*(\beta, p) + \mu_2 r_2^*(\beta, p) \quad (39)$$

We have the following lemma whose proof is provided in Appendix A:

Lemma 3. *$g(p)$ is monotone increasing, strictly concave function of p.*

Then, the optimization problem in (21) is equivalently stated as an optimization problem only in terms of p_i as follows:

$$\begin{aligned}\max_{\mathbf{p}} \quad & \sum_{i=1}^{N+1} g(p_i)\ell_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_i\ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad \forall k \\ & \sum_{i=1}^{k} p_i\ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max}\right)^+, \quad \forall k \\ & p_k \geqslant 0, \quad \forall k \end{aligned} \quad (40)$$

The optimization problem in (40) is a convex optimization problem. The objective function is strictly concave by Lemma 3 and the feasible set is a convex set.

Following the steps for finding the optimal policy in non-fading scalar broadcast channels, and as the objective function in (40) is concave, we obtain an important characteristic of optimal policies that achieve the boundary of $\mathcal{D}(T)$ of parallel broadcast channels.

Lemma 4. *For any point on the boundary of $\mathcal{D}(T)$ of parallel broadcast channels, the optimal total transmit power allocation sequence is the same as the optimal single-user power allocation policy in the scalar case.*

With Lemma 4 and the preceding findings, we obtain the full structure of a point on the boundary of the maximum departure region $\mathcal{D}(T)$. We first calculate the total power allocated for each receiver using the tightest curve approach in [4,5] if $E_{max} = \infty$, or the feasible tunnel approach in [6] or the directional water-filling algorithm in [7] if E_{max} is finite. As a result, we get the sequence of total powers allocated at each time epoch, $\{p_i^*\}_{i=1}^{N+1}$. Then, we divide each p_i^* as $p_{i1}^* = \beta_i^* p$ and $p_{i2}^* = (1-\beta_i^*)p$ allocated to the two parallel broadcast channels by means of the water-filling solution described in (37) and (38). With this, we get the power shares for each parallel channel p_{i1}^* and p_{i2}^* as well as the corresponding power shares of user 1 in each parallel channel $\alpha_1^*(p_{i1}^*)$ and $\alpha_2^*(p_{i2}^*)$. Then, (B_1^*, B_2^*) point that corresponds to the priority coefficients μ_1 and μ_2 is

$$\begin{aligned} B_1^* = \sum_{i=1}^{N+1} & \frac{1}{2}\log\left(1 + \frac{\alpha_1^*(p_{i1}^*)p_{i1}^*}{\sigma_{11}^2}\right)\ell_i \\ & + \frac{1}{2}\log\left(1 + \frac{\alpha_2^*(p_{i2}^*)p_{i2}^*}{(1-\alpha_2^*(p_{i2}^*))p_{i2}^* + \sigma_{12}^2}\right)\ell_i \end{aligned} \quad (41)$$

$$\begin{aligned} B_2^* = \sum_{i=1}^{N+1} & \frac{1}{2}\log\left(1 + \frac{(1-\alpha_2^*(p_{i2}^*))p_{i2}^*}{\sigma_{22}^2}\right)\ell_i \\ & + \frac{1}{2}\log\left(1 + \frac{(1-\alpha_1^*(p_{i1}^*))p_{i1}^*}{\alpha_1^*(p_{i1}^*)p_{i1}^* + \sigma_{21}^2}\right)\ell_i \end{aligned} \quad (42)$$

6. The optimal policy for fading broadcast channels

We now consider the fading broadcast channel. In order to solve (26), we first optimize the cost function in the ith epoch over α_i for a given total transmit power p_i. Consider the single-variable optimization problem in α for a given p:

$$\max_{0\leqslant\alpha\leqslant 1} \mu_1 r_1(n_1, n_2, \alpha, p) + \mu_2 r_2(n_1, n_2, \alpha, p) \quad (43)$$

The optimal solution of (43) is denoted by $\alpha = \alpha^*(n_1, n_2, p)$. Assume $n_1 < n_2$ and let $\mu = \mu_2/\mu_1$. If $1 < \mu < \frac{n_2}{n_1}$, $\alpha^*(n_1, n_2, p)$ is expressed as:

$$\alpha^*(n_1, n_2, p) = \begin{cases} 1, & 0 \leqslant p \leqslant \frac{\mu n_1 - n_2}{1-\mu} \\ \frac{1}{p}\frac{\mu n_1 - n_2}{1-\mu}, & p \geqslant \frac{\mu n_1 - n_2}{1-\mu} \end{cases} \quad (44)$$

In the extreme cases, $\alpha^*(n_1, n_2, p) = 1$ if $\mu \leqslant 1$ and $\alpha^*(n_1, n_2, p) = 0$ if $\mu \geqslant \frac{n_2}{n_1}$. If the order of noises is the other way, i.e., if $n_2 < n_1$, by changing the definition of μ as $\mu = \frac{\mu_1}{\mu_2}$,

$$\alpha^*(n_1, n_2, p) = \begin{cases} 0, & 0 \leqslant p \leqslant \frac{\mu n_2 - n_1}{1-\mu} \\ 1 - \frac{1}{p}\frac{\mu n_1 - n_2}{1-\mu}, & p \geqslant \frac{\mu n_2 - n_1}{1-\mu} \end{cases} \quad (45)$$

We define

$$h(n_1, n_2, p) \triangleq \mu_1 r_1(n_1, n_2, \alpha^*, p) + \mu_2 r_2(n_1, n_2, \alpha^*, p)$$

We have the following due to Lemma 2.

Lemma 5. *$h(n_1, n_2, p)$ is monotone increasing, strictly concave function of p given n_1 and n_2.*

In particular, $h(n_1, n_2, p)$ has a continuous monotone decreasing first derivative: for $n_1 < n_2$, whenever $\alpha^*(n_1, n_2, p) = 1$, the derivative is $\frac{\mu_1}{p+n_1}$ and otherwise, it is $\frac{\mu_2}{p+n_2}$. Similarly, if $n_2 < n_1$, whenever $\alpha^*(n_1, n_2, p) = 0$, the derivative is $\frac{\mu_2}{p+n_2}$ and otherwise, it is $\frac{\mu_1}{p+n_1}$. Hence, by first optimizing over α_i in (26), we obtain the following convex optimization problem over the total power sequence $\{p_i\}$:

$$\begin{aligned}\max_{\mathbf{p}} \quad & \sum_{i=1}^{N+K+1} h(n_{1i}, n_{2i}, p_i)\ell_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_i\ell_i \leqslant \sum_{i=0}^{k-1} E_i, \quad \forall k \\ & \sum_{i=1}^{k} p_i\ell_i \geqslant \left(\sum_{i=0}^{k} E_i - E_{max}\right)^+, \quad \forall k \\ & p_k \geqslant 0, \quad \forall k \end{aligned} \quad (46)$$

The optimization problem in (46), and hence the one in (26), has a unique optimal solution.

We define the Lagrangian for the problem in (46) as,

$$\begin{aligned}\mathcal{L} = & \sum_{i=1}^{N+K+1} h(n_{1i}, n_{2i}, p_i)\ell_i - \sum_{j=1}^{N+K+1} \lambda_j\left(\sum_{i=1}^{j} p_i\ell_i - \sum_{i=0}^{j-1} E_i\right) \\ & - \sum_{j=1}^{N+K+1} \kappa_j\left(\left(\sum_{i=0}^{j} E_i - E_{max}\right)^+ - \sum_{i=1}^{j} p_i\ell_i\right) + \sum_{i=1}^{N+K+1} \eta_i p_i \end{aligned} \quad (47)$$

The first order condition on the Lagrangian is

$$\frac{d}{dp_i} h(n_{1i}, n_{2i}, p_i) = \sum_{j=i}^{N+K+1} \lambda_j - \sum_{j=i}^{N+K+1} \kappa_j - \eta_i \quad (48)$$

The complimentary slackness conditions are:

$$\lambda_j\left(\sum_{i=1}^{j} p_i\ell_i - \sum_{i=0}^{j-1} E_i\right) = 0, \quad \forall j \quad (49)$$

$$\kappa_j\left(\left(\sum_{i=0}^{j} E_i - E_{max}\right)^+ - \sum_{i=1}^{j} p_i\ell_i\right) = 0, \quad \forall j \quad (50)$$

$$\eta_j p_j = 0, \quad \forall j \quad (51)$$

It follows that the optimal total power in epoch i is given by

$$p_i^* = \mu_{u_i}\left[v_i - \frac{1}{\mu_{u_i}h_{u_i}}\right]^+ \quad (52)$$

where the water level in epoch i, v_i, is

$$v_i = \frac{1}{\sum_{j=i}^{N+K+1}\lambda_j - \sum_{j=i}^{N+K+1}\kappa_j} \quad (53)$$

The index u_i is uniquely determined by the given μ_1, μ_2, n_1, n_2 and E_i. In particular, u_i is 1 if the derivative of $h(n_1, n_2, p)$ at the allocated power p_i^* in (52) is $\frac{\mu_1}{p+n_1}$ and it is 2 otherwise. For $\frac{\mu_2}{\mu_1} \leqslant \min_i\{\frac{n_{1i}}{n_{2i}}, \frac{n_{2i}}{n_{1i}}\}$, $u_i = 1$ for all i and all the power is allocated to the first user only. If $\frac{\mu_2}{\mu_1} \geqslant \max_i\{\frac{n_{1i}}{n_{2i}}, \frac{n_{2i}}{n_{1i}}\}$, $u_i = 2$ for all i and all the power is allocated to the second user. For the remaining values of $\frac{\mu_2}{\mu_1}$, both users may be allocated power in some epoch.

Note that the slackness variables λ_i and κ_i are zero in between two energy harvesting instants as the energy causality and no-energy-overflow constraints are never violated except possibly at the energy arriving instants. Therefore, the water level v_i is the same for all epochs in between two energy harvesting instants. When $E_{max} = \infty$, for any epoch i, the optimum water level v_i is monotonically increasing, i.e., $v_{i+1} \geqslant v_i$ as $\kappa_j = 0$ in this case. If some energy is transferred from epoch i to $i+1$, then $v_i = v_{i+1}$.

For finite E_{max} case, the solution is found by a directional water-filling algorithm [7], which we describe next. The directional water-filling algorithm requires walls at the points of energy arrival, with right permeable water taps in each wall which allows at most E_{max} amount of water to flow, as shown in Fig. 6. First, the taps are kept off and transfer from one epoch to the other is not allowed. Each incoming energy E_i is spread in the time interval till the next energy arrival time and the water level is calculated. The main difficulty arises due to the fact that the index u_i is not known a priori. If a sequence of u_i is assumed, the resulting water levels and power allocation should be compatible with (45) and there exists a unique u_i^* sequence that is compatible with (45). The resulting water levels v_i can be found by the water-filling algorithm in [18] or the greedy water-filling algorithm in [19]. The water levels when each right permeable tap is turned on will be found allowing at most $E_{max} - E_i$ amount of energy transfer from the past epochs to the epochs which start with arrival of E_i provided that the initial water level in epoch $i-1$ is higher than that in epoch i. This is due to the fact that the slackness variable κ_i is not active if energy transfer from past to the future is less than $E_{max} - E_i$. If κ_i is not active, water level v_i in the past should be less than or equal to the water level v_j in the future. As $\lambda_i = 0$ if an energy arrival does not occur at epoch i, we conclude that the incoming energy should be spread till the time next energy arrives. Optimal power allocation p_i^* is then calculated by plugging the resulting water levels into (52). We note that the water level is scaled by different priority coefficients μ_{u_i} to yield the energy consumed at each epoch. Individual power shares are then found via (45). The optimal solution is unique unless $n_{1i} = n_{2i}$ for some epoch i. If $n_{1i} = n_{2i}$ for some epoch i, the optimal policy when $\mu_1 = \mu_2$ is any policy formed by time-sharing between giving strict priority to one of the users at that

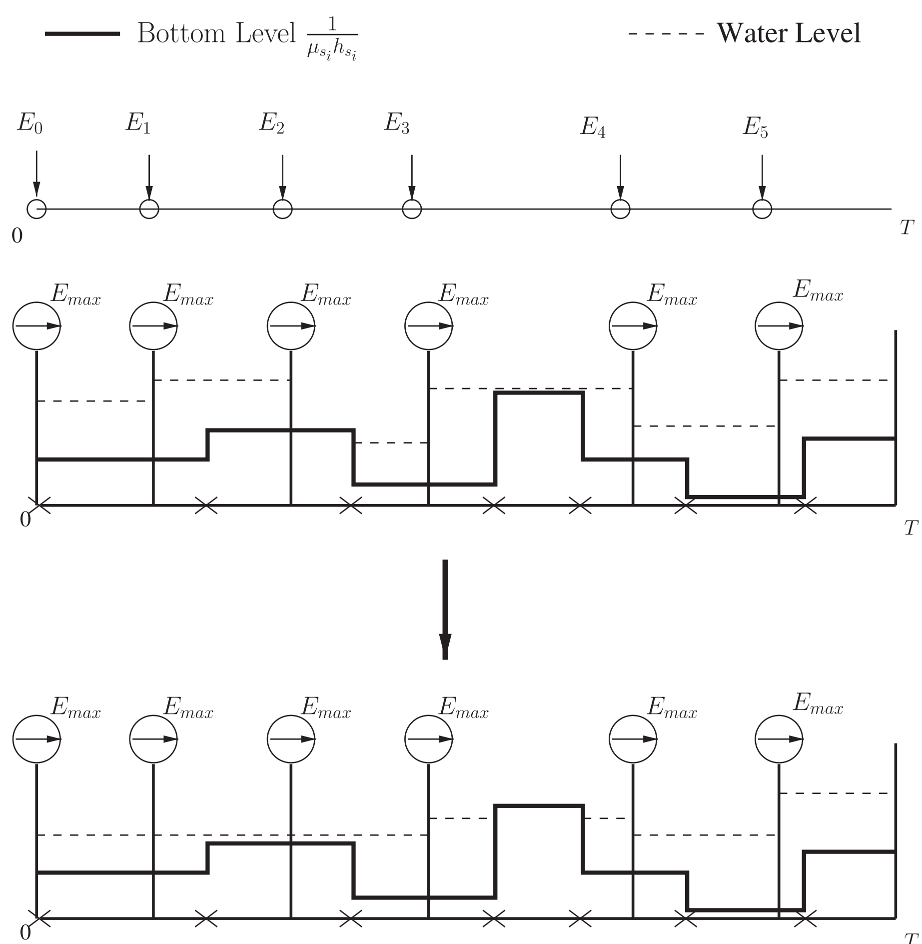

Fig. 6. Directional water-filling algorithm.

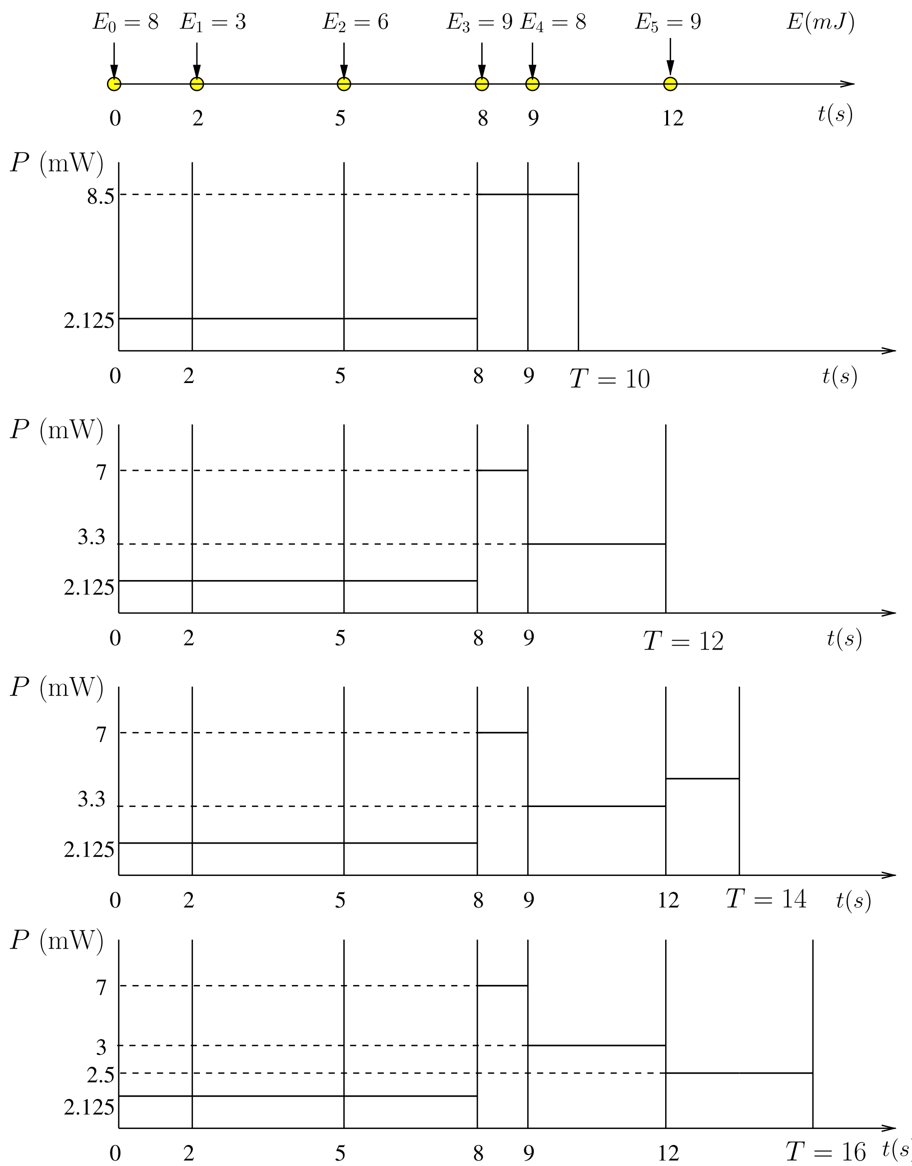

Fig. 7. Energy arrivals occur at $[2, 5, 8, 9, 12]$ s with amounts $[3, 6, 9, 8, 9]$ mJ and the initial energy in the battery at time zero $E_0 = 8$ mJ. The optimal total power sequence for $T = 10$ s, $T = 12$ s, $T = 14$ s and $T = 16$ s.

epoch. In this case, the sum throughput optimal points of $\mathcal{D}(T)$ form a line.

An example run of the algorithm is shown in Fig. 6, for a case of 12 epochs. Five energy arrivals occur during the communication session, in addition to the energy available at time $t = 0$. We observe that the water level equalizes in epochs 1, 2, 3, 4, 5. No power is transmitted in epoch 7, since $\frac{1}{\mu_{u_i} h_{u_i}}$ is too high. The energy arriving at the beginning of epoch 6 cannot flow left due to energy causality constraints, which are ensured by right permeable taps. We observe that the excess energy in epochs 6, 7 and 8 cannot flow right, due to the E_{max} constraint at the beginning of epoch 9.

We remark here that the optimal policy strongly depends on the priority coefficients μ_1, μ_2 of the users in contrast to the non-fading and parallel broadcast channels in which the optimal total power sequence is independent of μ_1 and μ_2. In particular, the bottom level of the directional water-filling is determined by the particular values of μ_1 and μ_2. If the user priorities are identical, i.e., $\mu_1 = \mu_2$, then the optimal policy is equal to the single-user transmission policy for the user with the best channel at each epoch. The power allocation is found by applying the directional water-filling algorithm in [7] by selecting the bottom level in Fig. 6 as $\frac{1}{\max\{h_{1i}, h_{2i}\}}$.

We finally remark that our analysis can be extended for the case in which the transmitter sends messages over parallel broadcast channels with time-varying channel gains. For given channel gains, the share variables α_1, α_2 and β are defined as in (19) and (20) and after optimizing the weighted sum of rates over the share variables as in (43) we obtain a strictly concave function of power due to Lemma 5. Using similar convex optimization tools, we conclude that the solution is unique and it is found by a generalized directional water-filling algorithm.

7. Numerical illustrations

In this section, we provide numerical illustrations for the maximum departure region over parallel and fading broadcast channels. We start with parallel channels and then consider fading broadcast channels.

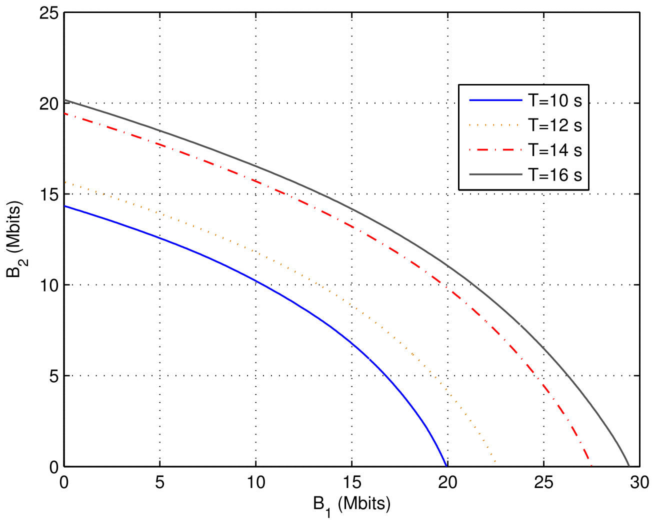

Fig. 8. The maximum departure region for the parallel broadcast channel under the given energy arrivals for various T.

7.1. Parallel broadcast channels

We consider a band-limited two-user AWGN broadcast channel with two parallel channels operating with a bandwidth of $W = 1$ MHz and under noise power spectral density $N_0 = 10^{-19}$ W/Hz. In the first channel, the path loss between the transmitter and receiver 1 is $c_{11} = 100$ dB and between the transmitter and receiver 2 is $c_{21} = 105$ dB. We have

$$r_{11} = W\log_2\left(1 + \frac{\alpha_1 c_{11}\beta P 10^{-3}}{N_0 W}\right)$$
$$= \log_2\left(1 + \frac{\alpha_1 \beta P}{n_{11}}\right) \text{ Mbps} \tag{54}$$

and

$$r_{21} = W\log_2\left(1 + \frac{(1-\alpha_1)c_{21}\beta P 10^{-3}}{\alpha_1 c_{21}\beta P 10^{-3} + N_0 W}\right)$$
$$= \log_2\left(1 + \frac{(1-\alpha_1)\beta P}{\alpha_1 \beta P + n_{21}}\right) \text{ Mbps} \tag{55}$$

where $n_{11} = 1$ and $n_{21} = 10^{0.5}$. The second parallel channel has path loss coefficients $c_{12} = 107$ dB and $c_{22} = 103$ dB and the resulting rate expressions are

$$r_{12} = \log_2\left(1 + \frac{\alpha_2(1-\beta)P}{(1-\alpha_2)(1-\beta)P + n_{12}}\right) \text{ Mbps} \tag{56}$$

$$r_{22} = \log_2\left(1 + \frac{(1-\alpha_2)(1-\beta)P}{n_{22}}\right) \text{ Mbps} \tag{57}$$

where $n_{12} = 10^{0.7}$ and $n_{22} = 10^{0.3}$.

We assume that the battery capacity is $E_{max} = 10$ mJ and the energy arrivals occur at time instants $t_1^e = 2$ s, $t_2^e = 5$ s, $t_3^e = 8$ s, $t_4^e = 9$ s, $t_5^e = 12$ s with amounts $E_1 = 3$ mJ, $E_2 = 6$ mJ, $E_3 = 9$ mJ, $E_4 = 8$ mJ, $E_5 = 9$ mJ. The battery energy at time $t = 0$ s is $E_0 = 8$ mJ. We show the optimal total transmit power sequences for $T = 10$ s, $T = 12$ s, $T = 14$ s and $T = 16$ s in Fig. 7. Initial energy in the battery and the first two energy arrivals are spread till $t = 8$ s. However, at most 2 mJ energy can flow from the time interval [8,9] s to the future as the finite battery constrains the energy flow. For example, for $T = 10$ s, only 0.5 mJ energy is transferred from [8,9] s interval while for $T = 12$ s, 2 mJ limit is hit and the power in [8,9] s is kept at 7 mJ (which leads to 7 mW power in that interval). Similarly, at most 1 mJ energy can flow from [9,12] s interval to the future. This leads to a non-monotonic total transmit power sequence as opposed to the $E_{max} = \infty$ case. We plot the resulting maximum departure regions in Fig. 8. Note that the maximum departure regions are strictly convex for all T and monotone in T. We observe that the gap between the regions for different T

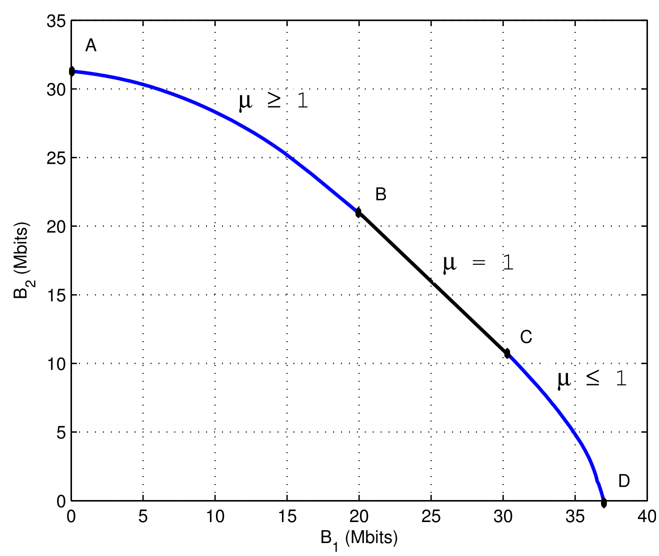

Fig. 10. The maximum departure region for the fading broadcast channel under the given energy arrival and fading profiles at $T = 14$ s.

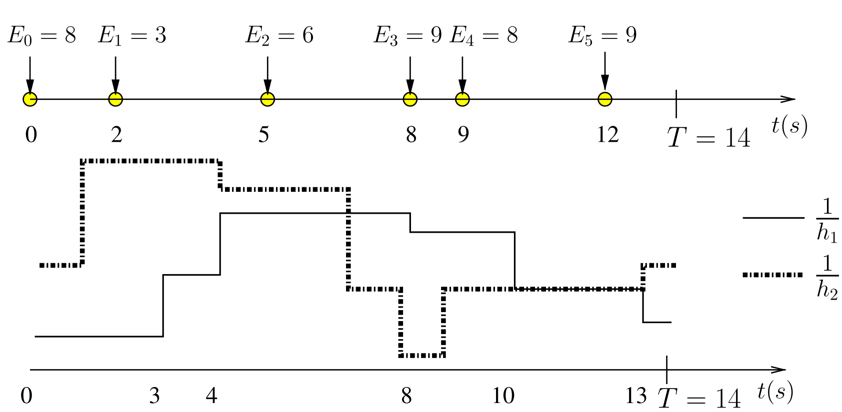

Fig. 9. The energy and fading profiles for the fading broadcast channel.

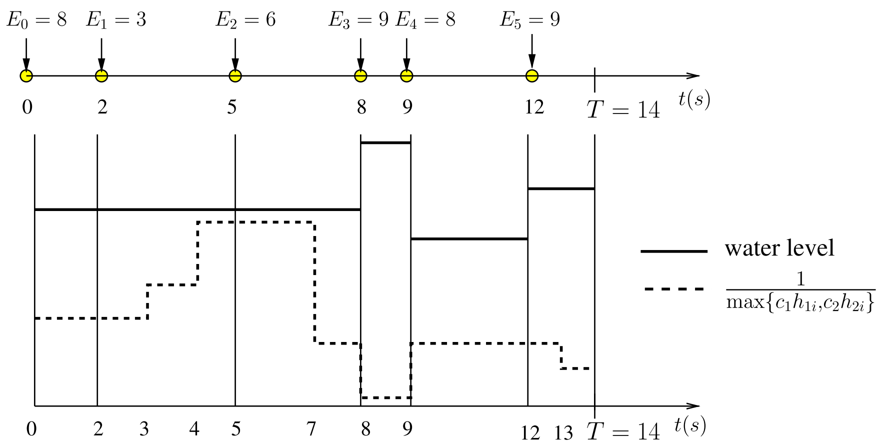

Fig. 11. The sum throughput optimal policy obtained by directional water-filling.

increases in the passage from $T = 12$ s to $T = 14$ s since an energy arrival occurs at $t = 12$ s. This is reminiscent of the fact that in a single-user energy harvesting system, the rate of increase of the maximum departure curve is infinite at energy harvesting instants as observed in [7].

7.2. Fading broadcast channels

We consider a band-limited AWGN broadcast channel with bandwidth $W = 1$ MHz and noise power spectral density $N_0 = 10^{-19}$ W/Hz. The path loss between the transmitter and receiver 1 is $c_1 = 100$ dB and between the transmitter and receiver 2, is $c_2 = 105$ dB. In addition, the channel fading coefficients h_1 and h_2 vary during the transmission. We have

$$r_1 = W\log_2\left(1 + \frac{\alpha c_1 h_1 P 10^{-3}}{(1-\alpha) c_1 h_1 P 10^{-3} \mathbf{1}(c_1 h_1 < c_2 h_2) + N_0 W}\right) \quad (58)$$

$$= \log_2\left(1 + \frac{\alpha P}{(1-\alpha) P \mathbf{1}(c_1 h_1 < c_2 h_2) + n_1}\right) \text{Mbps} \quad (59)$$

and similarly

$$r_2 = \log_2\left(1 + \frac{(1-\alpha) P}{\alpha P \mathbf{1}(c_1 h_1 > c_2 h_2) + n_2}\right) \text{Mbps} \quad (60)$$

where $n_1 = \frac{1}{h_1}$ and $n_2 = \frac{10^{0.5}}{h_2}$. The fading profile, $\mathbf{h}_i = (h_{1i}, h_{2i})$ where i is the time index and both entries are in dB, is $\mathbf{h}_1 = (7,4)$, $\mathbf{h}_2 = (7,2)$, $\mathbf{h}_3 = (2,2)$, $\mathbf{h}_4 = (-1,3)$, $\mathbf{h}_5 = (-1,8)$, $\mathbf{h}_6 = (1,13)$, $\mathbf{h}_7 = (1,8)$, $\mathbf{h}_8 = (3,8)$ and $\mathbf{h}_9 = (5,7)$ at time instants $t_1^f = 0$ s, $t_2^f = 1$ s, $t_3^f = 3$ s, $t_4^f = 4$ s, $t_5^f = 7$ s, $t_6^f = 8$ s, $t_7^f = 10$ s, $t_8^f = 11$ s. We show the energy and fading profiles in Fig. 9. In particular, the fading profiles in Fig. 9 are the inverted overall channel gains of the users, i.e., the path loss times fading coefficients.

We plot the maximum departure region corresponding to the given energy and channel profiles for $T = 14$ s in Fig. 10. There are four critical points of the maximum departure region, A, B, C and D, as indicated in Fig. 10. At point A, all the power is allocated for the transmission of user 1 and no data is transmitted for user 2; point D is vice versa. At points B and C, the priorities of the users are equal, i.e., $\mu = \frac{\mu_2}{\mu_1} = 1$. For the points to the left of B, $\mu \geqslant 1$ and for the points to the right of C, $\mu \leqslant 1$. The total power allocation at points A and D are found by single-user directional water-filling in [7] with the bottom level selected as $\frac{1}{c_1 h_{1i}}$ and $\frac{1}{c_2 h_{2i}}$, respectively. Moreover, the total power allocation at the sum throughput optimal policies (points B and C) is found by single-user directional water-filling with the bottom level selected as $\frac{1}{\max\{c_1 h_{1i}, c_2 h_{2i}\}}$. In Fig. 11, we show the total power allocation of the sum throughput optimal policies corresponding to the time-sharing between points B and C in Fig. 10. Note that the total power allocation is not affected by the choice of the index u_i at epochs i in which $c_1 h_{1i} = c_2 h_{2i}$. As $c_1 h_{1i} = c_2 h_{2i}$ holds for some i, time sharing between these users in these epochs does not violate optimality for $\mu = 1$. Therefore, the boundary of the maximum departure region includes a line segment with a slope of -45°. We remark that if $h_{1i} \neq h_{2i}$ for all i, the boundary of the maximum departure region does not include a line segment, i.e., it is strictly convex. In an ergodic setting with continuous fading distributions, under some mild conditions, the probability that $c_1 h_{1i} = c_2 h_{2i}$ for some i is zero and therefore the ergodic capacity region is strictly convex [18].

8. Conclusions

In this paper, we considered communication over parallel and fading broadcast channels with an energy harvesting rechargeable transmitter that has a finite-capacity battery. We characterized the region of bit departures by a deadline T in an off-line setting where changes in the energy and fading levels are known a priori at the transmitter. For parallel broadcast channels, we showed that the optimal total power allocation sequence is the same as that for the non-fading broadcast channel, which does not depend on the priorities of the users and equals the single-user optimal power allocation policy. The total power is split for the parallel channels in each interval separately. For fading broadcast channels, in contrast with non-fading broadcast channels, we showed that the optimal power allocation policy strongly depends on the priorities of the users and it is found by a specific directional water-filling algorithm. Finally, we provided illustrations for the maximum departure region for both parallel and fading broadcast channels.

Appendix A. Proof of Lemma 3

Continuity of $g(p)$ follows from the continuity of g_1 and g_2. In order to prove that $g(p)$ is strictly concave, we need to show the following

$$g(\lambda p_1 + (1-\lambda) p_2) > \lambda g(p_1) + (1-\lambda) g(p_2) \quad (61)$$

for all $0 < \lambda < 1$.

We define the following functions for each parallel channel:

$$g_1(p) \triangleq \max_{0\leqslant\alpha\leqslant 1} \frac{\mu_1}{2}\log_2\left(1+\frac{\alpha p}{\sigma_{11}^2}\right)+\frac{\mu_2}{2}\log_2\left(1+\frac{(1-\alpha)p}{\alpha p+\sigma_{21}^2}\right) \tag{62}$$

$$g_2(p) \triangleq \max_{0\leqslant\alpha\leqslant 1} \frac{\mu_1}{2}\log_2\left(1+\frac{\alpha p}{(1-\alpha)p+\sigma_{12}^2}\right) + \frac{\mu_2}{2}\log_2\left(1+\frac{(1-\alpha)p}{\sigma_{22}^2}\right) \tag{63}$$

We first note that both $g_1(p)$ and $g_2(p)$ are continuous, strictly concave functions of p due to Lemma 2 in [11].

$g(p)$ in Lemma 3 can be expressed in terms of $g_1(p)$ and $g_2(p)$ as follows:

$$g(p) = \max_{0\leqslant\beta\leqslant 1} g_1(\beta p) + g_2((1-\beta)p) \tag{64}$$

Therefore, for any $0 \leqslant \beta \leqslant 1$, we have

$$g(p) \geqslant g_1(\beta p) + g_2((1-\beta)p) \tag{65}$$

We now prove the strict concavity. Let p_1 and p_2 be given. Let β_1 be the solution of (64) when $p = p_1$ and β_2 be the solution when $p = p_2$. Then,

$$g(p_1) = g_1(\beta_1 p_1) + g_2((1-\beta_1)p_1) \tag{66}$$

$$g(p_2) = g_1(\beta_2 p_2) + g_2((1-\beta_2)p_2) \tag{67}$$

For any $0 < \lambda < 1$, we have

$$\begin{aligned} &g(\lambda p_1 + (1-\lambda)p_2) \\ &\geqslant g_1(\lambda\beta_1 p_1 + (1-\lambda)\beta_2 p_2) + g_2(\lambda(1-\beta_1)p_1 + (1-\lambda)(1-\beta_2)p_2) && (68) \\ &> \lambda g_1(\beta_1 p_1) + (1-\lambda)g_1(\beta_2 p_2) + \lambda g_2((1-\beta_1)p_1) + (1-\lambda)g_2((1-\beta_2)p_2) && (69) \\ &= \lambda g(p_1) + (1-\lambda)g(p_2) && (70) \end{aligned}$$

The inequality in (68) is by evaluating (65) for $p = \lambda p_1 + (1-\lambda)p_2$ and $\beta = \frac{\lambda\beta_1 p_1 + (1-\lambda)\beta_2 p_2}{\lambda p_1 + (1-\lambda)p_2}$. (69) is due to the concavity of $g_1(p)$ and $g_2(p)$ and (70) is a rearrangement of (69). This proves the strict concavity of $g(p)$.

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