Optimal Collaborative Sensing Scheduling with Energy Harvesting Nodes

Jing Yang
Department of Electrical Engineering
University of Arkansas, Fayetteville, AR 72701
jingyang@uark.edu

***Abstract*—In this paper, we consider a collaborative sensing scenario where sensing nodes are powered by energy harvested from environment. We assume that in each time slot, the utility generated by sensing nodes is a function of the number of the active sensing nodes in that slot. Under the energy causality constraint at every sensor, our objective is to develop a collaborative sensing scheduling for the sensors such that the time average utility is maximized. We consider an offline setting, where the energy harvesting profile over duration $[0, T-1]$ for each sensor is known beforehand. Under the assumption that the utility function is concave over $\mathbb{Z}_+$, we first propose an algorithm to identify the number of active sensors in each slot. The obtained scheduling structure has a "majorization" property. We then propose a procedure to construct a collaborative sensing policy with the identified structure. The obtained sensing scheduling is proved to be optimal.**

I. Introduction

Sensor networks equipped with energy harvesting devices have attracted great attentions recently. Compared with conventional sensor networks powered by batteries, the energy harvesting abilities of the sensor nodes make sustainable and environment-friendly sensor networks possible. Such renewable energy supply feature also necessitates a completely different approach to energy management.

Under an energy harvesting setting, energy management schemes have been developed to optimize communication related metrics, such as channel capacity, transmission delay or network throughput [1]–[8], and signal processing related performance metrics, such as estimation mean square error, detection delay, false alarm probability [9], [10].

In this paper, we focus on the design of a collaborative sensing scheme in a sensor network powered by energy harvested from the environment. Our motivation is a collaborative sensing scenario where multiple sensors are deployed to monitor the spectrum usage in an area. While collaborative sensing schemes have been well studied under a conventional battery-powered setting, the optimal sensing scheduling for rechargeable sensing nodes has not been studied before. Our objective is to coordinate the sensing actions among multiple sensor nodes in a way that the time average sensing performance (such as the probability of detection error) is optimized. Our primary constraint is the energy causality constraint at each sensor node.

Specifically, we assume that a sensor takes a unit of energy to sense the nature and send its observation to a fusion center (FC). Sensors cannot perform the sensing task when there is not sufficient energy in its battery. The FC combines the observations collected from sensors and infer the underlying spectrum usage status. We assume that the inference performance is measured in terms of the *utility* generated by the observations. The utility generated in each slot is a function of the set of active sensors in that slot. Our objective is to select a subset of sensors to perform the sensing task in each time slot, such that the long-term average utility is optimized, while the energy constraint at each individual sensor is satisfied at every time slot. The problem has a combinatorial nature and is hard to solve in general. The randomness of the energy harvesting processes at sensors make the problem even more challenging.

To make the problem tractable, we assume that the utility function is symmetric with respect to sensors, i.e., it is a function of the total number of active sensors in each slot. In addition, we assume that it is a concave function defined over $\mathbb{Z}_+$. Under such assumptions, we show that the optimal sensing scheduling has a "majorization" structure, i.e., the number of active sensors in each slot should be as even as possible, subject to the energy causality constraints at individual sensors. We propose an algorithm to identify the optimal number of active sensors in each slot, and construct a sensing scheduling with the identified subset sizes.

We point out that a similar "majorization" scheduling structure has been observed in throughput optimization problems with energy harvesting transmitters [1], [4], [6]. In [1], the optimal transmission policy for a single transmitter under the given energy causality constraint is to equalize the transmit power as much as possible. The "majorization" structure of the solution is due to the concavity of the function $r = \frac{1}{2}\log(1 + P)$. However, there are fundamental differences between problem studied in this paper and [1]. The optimization problem in this paper is to select a subset of sensors in each slot, and each selected sensor consumes a unit of energy for sensing, while in [1], the objective is to vary the power to maximize the throughput. The latter is formulated as a convex optimization problem, while the former has a combinatorial nature, and in general cannot be solved through convex optimization.

II. System Model and Problem Formulation

In this paper, we consider a sensor network consisting of N sensors (randomly) distributed in an area. Each sensor node is powered by energy harvested from ambient environment. We assume that each sensor node has an energy queue to store the harvested energy. The energy queue has a maximum storage capacity E_{max}. For now, we consider the case where $E_{max} = +\infty$. The energy queue at each sensor is replenished randomly and consumed by taking observations and transmitting them to a fusion center (FC). We assume that a unit amount of energy is required for one sense-and-transmit operation.

We consider a time-slotted system. In time slot t, a subset of sensors, denoted as $\mathcal{C}_t$, is selected to sense the environment, and transmit their observations to the FC. We assume a sensor can make at most one observation in each slot. The FC then combines the observations for inference. The utility generated by those observations is a function of $\mathcal{C}_t$, denoted as $f(\mathcal{C}_t)$. The total sensing utility over duration $[1, T]$ is simply the sum of the utilities generated in each slot in $[1, T]$. We make the following assumptions on the utility function $f(\mathcal{C}_t)$.

Assumptions 1

(0) *$f(\mathcal{C})$ is a function of the size of $\mathcal{C}$, i.e., $f(\mathcal{C}) = f(|\mathcal{C}|)$.*
(i) *$f(\emptyset) = f(0) = 0$.*
(ii) *$f(m)$ is monotonically increasing in m.*
(iii) *$f(m+1) + f(m-1) < 2f(m)$ for $m \in \mathbb{Z}_+$.*

Assumption 1-(0) implies that $f(\mathcal{C})$ is symmetric with respect to sensor nodes. By imposing this assumption, we essentially ignore the differences in contributions from different sensing nodes, and focus on the impact of the total number of collected observations on the sensing performance. Assumption 1-(i)(ii) are natural assumptions, as Assumption 1-(i) indicates that no utility can be gained if no observation has been made, and Assumption 1-(ii) means that the utility function increases as more observations are collected. Assumption 1-(iii) essentially means that $f(m)$ is a concave function defined over $\mathbb{Z}_+$.

Let $E_i(t)$ denote the amount of energy remaining in the battery of node i at the beginning of time slot t, $A_i(t)$ be the amount of harvested energy at node i during slot t. Without loss of generality, we assume $A_i(t) \in \mathbb{Z}_+$. Then, the energy queue evolves according to

$$E_i(t+1) = E_i(t) - \mathbf{1}_{i \in \mathcal{C}_t} + A_i(t), \tag{1}$$

where $\mathbf{1}_x$ is an indicator function, i.e., it equals one if x is true, and it equals zero otherwise. Since an observation cannot be made if $E(t) < 1$, we impose the following energy constraints

$$E_i(t) \geq \mathbf{1}_{i \in \mathcal{C}_t}, \qquad \forall t, i. \tag{2}$$

We consider an offline setting, i.e., $A_i(t), t = 0, 1, \ldots, T-1$ are known beforehand. Our objective is to select the subset of sensors $\mathcal{C}_t$ to perform the sensing task in each time slot t, such that the time average utility generated over $[1, T]$ is maximized. Such scheduling must satisfy the energy constraint for each individual sensor at every time slot. Thus, the optimization problem is formulated as

$$\max_{\{\mathcal{C}_t\}} \qquad \frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t) \quad \text{s.t. } (1)-(2) \tag{3}$$

III. The Optimal Sensing Scheduling

The optimization problem in (3) has a combinatorial nature, and is in general hard to solve. However, with Assumption 1, we show that the optimal solution has a "majorization" structure, which can be exploited to obtain the optimal sensing scheduling explicitly. In this section, we first describe a procedure to determine the structure of the optimal scheduling, and then construct a scheduling policy explicitly with the obtained structure.

A. Identify a Majorization Scheduling Structure

First, we note the energy harvesting profile $\{A_i(t)\}_t$ for sensor i imposes constraints on the total number of time slots that a sensor be active over $[1, t], \forall t$. Let $B_i(t) = \sum_{j=0}^{t-1} A_i(j)$ be the total amount of energy harvested upto the beginning of time slot t. Then, the total number of time slots that a sensor can be *active* over $[1, t]$ is upper bounded by $B_i(t)$. However, we note that since at most one unit of energy can be spent in each slot, a sensor may not be able to spend all of the $B_i(t)$ units of energy over $[1, t]$. To provide a tight bound on the total number of times slots that an sensor can be active up to t, we introduce another quantity $S_i(t)$, which is defined recursively as follows

$$S_i(0) = 0, \quad \forall i \tag{4}$$

$$S_i(t) = \min\{S_i(t-1) + 1, B_i(t)\}, \quad \forall i, t \tag{5}$$

Based on this definition, we have

$$\sum_{j=1}^{t} \mathbf{1}_{i \in \mathcal{C}_t} \leq S_i(t), \quad \forall i \tag{6}$$

Sum up the inequalities in (6) over i, we get

$$\sum_{i=1}^{N}\sum_{j=1}^{t} \mathbf{1}_{i \in \mathcal{C}_t} \leq \sum_{i=1}^{N} S_i(t) := S(t)$$

which is equivalent to

$$\sum_{j=1}^{t} |\mathcal{C}_t| \leq S(t), \quad \forall t \tag{7}$$

Eqn. (7) imposes a constraint on the accumulative number of observations the FC can collect up to time slot t. Due to the concavity of the utility function $f(\mathcal{C}_t)$ in $|\mathcal{C}_t|$, intuitively, to maximize the objective function in (3), we should equalize $\{|\mathcal{C}_t|\}_t$ as much as possible, under the constraints in (6) for each individual sensor. While handling N individual constraints simultaneously is too complicated, in the following, we equalize $\{|\mathcal{C}_t|\}_t$ under the sum constraint (7) only. In general, the solution obtained with such relaxation may not be feasible when individual constraints are imposed. However, as we will show in Sec. III-B, the $\{|\mathcal{C}_t|\}_t$ obtained under constraint (7) is always feasible.

The procedure to equalize $\{|\mathcal{C}_t|\}_t$ under the constraints in (7) is provided in Algorithm 1. Starting with $n = 0$, eqn. (8) calculates the average number of active nodes in each slot over $[1, t]$ assuming constraint (7) is tight at t, and pick the minimum as the *tentative* size of $\mathcal{C}_t$ for $1 \leq t \leq t_1$. Repeating this procedure until $t_n = T$, we obtain a sequence of *tentative* sizes for $\{\mathcal{C}_t\}_t$. The way we obtain the sequence implies that this is the most equalized scheduling structure under constraint (7), with equality met at time T.

However, the *tentative* size of $\mathcal{C}_t$ given by Algorithm 1 may not be an integer, which cannot be achieved since we cannot let a non-integer number of nodes be active in a slot. In order to obtain a valid scheduling, we adjust the *tentative* size according to (10). By rounding the *tentative* size down and up in this way, we keep the total number of observations collected over $[t_{n-1}+1, t_n]$ the same, and get rid of the potential problem caused by the non-integer number of active nodes. Intuitively, this is the most equalized valid scheduling structure we can have.

Algorithm 1 An algorithm to equalize $\{|\mathcal{C}_t|\}_{t=1}^T$

Input: $\{S(t)\}_{t=1}^T$.
Initialization: $n = 0$, $t_0 = 0$.
while $t_n < T$ **do**
  $n = n + 1$;
  Let

$$t_n = \arg \min_{t_{n-1} < t \leq T} \left\{ \frac{S(t) - S(t_{n-1})}{t - t_{n-1}} \right\} \tag{8}$$

$$r = S(t_n) - S(t_{n-1}) - (t_n - t_{n-1}) \left\lfloor \frac{S(t_n) - S(t_{n-1})}{t_n - t_{n-1}} \right\rfloor \tag{9}$$

$$c_t = \begin{cases} \left\lfloor \frac{S(t_n) - S(t_{n-1})}{t_n - t_{n-1}} \right\rfloor, & t_{n-1} < t \leq t_n - r \\ \left\lceil \frac{S(t_n) - S(t_{n-1})}{t_n - t_{n-1}} \right\rceil, & t_n - r < t \leq t_n \end{cases} \tag{10}$$

end while
Output: $\{c_t\}_{t=1}^T$.

B. Construct a Sensing Scheduling with $\{c_t\}_{t=1}^T$

With the scheduling structure, i.e., $\{c_t\}_{t=1}^T$, obtained in Algorithm 1, we aim to construct a sensing policy, such that the active number of nodes in slot t equals c_t exactly, and each individual energy constraint in (6) is satisfied.

We propose to construct the sensing scheduling in the following way. First, we obtain an initial scheduling by letting each sensor perform sensing in a greedy fashion. Specifically, we let each sensor node spend one unit of energy to take an observation whenever it has sufficient energy. By designing the sensing policy in this way, sensor i senses in time slot t whenever $S_i(t) - S_i(t-1) = 1$. Thus, we have exact $S(t) - S(t-1)$ active sensor nodes in slot t. Let $\mathcal{S}(t)$ be the set of active nodes in slot t, and $|\mathcal{S}(t)| := s_t$. Initially, $\mathcal{S}(t)$ includes all sensors with at least one unit of energy at the beginning of slot t under the greedy sensing policy.

Then, we adjust the initial scheduling by letting a subset of sensors postpone their sensing actions scheduled in certain time slots and sense with the saved energy in some later time slots. The rescheduling is coordinated in a way that exact c_t sensors are scheduled for sensing in time t.

The rescheduling is carried out progressively. Starting with $t = 1$, we search for the first t such that $s_t < c_t$, denoted as $\hat{t}$, and the first t such that $s_t > c_t$, denoted as $\bar{t}$. As we will see in Lemma 3, $\bar{t} < \hat{t}$. We then remove a subset of sensors from $\mathcal{S}(\bar{t})$, and add them to $\mathcal{S}(\hat{t})$, i.e., we let them be silent in time slot $\bar{t}$ and be active in $\hat{t}$. Since $\bar{t} < \hat{t}$, this does not violate the individual energy causality constraints in (6).

Recall that we assume each sensor can take at most one observation in each slot. Let $\delta = \min(s_{\bar{t}} - c_{\bar{t}}, c_{\hat{t}} - s_{\hat{t}})$. We then randomly remove δ sensors from $\mathcal{S}(\bar{t}) \backslash \mathcal{S}(\hat{t})$ and add them to $\mathcal{S}(\hat{t})$. If $\delta \neq c_{\hat{t}} - s_{\hat{t}}$, we search for the next $\bar{t}$ with $s_{\bar{t}} > c_{\bar{t}}$ and repeat the procedure. Once $s_{\hat{t}}$ becomes equal to $c_{\hat{t}}$, we search for the next $\hat{t}$ with $s_{\hat{t}} < c_{\hat{t}}$, repeat the procedure, until $t = T$.

In order to prove the feasibility of the described rescheduling procedure, we introduce the following Lemmas. The first two Lemmas can be easily proved based on Algorithm 1.

Lemma 1 $\sum_{j=1}^{t} c_j \leq S(t)$ *for* $1 \leq t \leq T$. *The equality holds if* $t \in \{t_n\}_n$.

Lemma 2 *If* $t_{n-1} < t_1 < t_2 \leq t_n$, *we must have either* $c_{t_1} = c_{t_2}$, *or* $c_{t_1} = c_{t_2} - 1$.

Lemma 3 *In each iteration of the rescheduling, if* $\sum_{t=1}^{\hat{t}} s_t = S(\hat{t})$, *we must have a)* $\bar{t} < \hat{t}$, *b)* $\sum_{t=1}^{\tau} s_t = S(\tau)$, $\forall \tau > \hat{t}$.

Proof: Part a) can be proved through contradiction. If $\bar{t} > \hat{t}$, based on the definition of $\bar{t}$ and $\hat{t}$, we have $\sum_{t=1}^{\hat{t}} s_t = \sum_{t=1}^{\hat{t}-1} c_t + s_t < \sum_{t=1}^{\hat{t}} c_t \leq S(\hat{t})$ where the last inequality follows from Lemma 1. It contradicts with the assumption of the Lemma. Thus, we must have $\bar{t} < \hat{t}$.

Part b) can be proved based on the observation that the rescheduling only involves $\{\mathcal{S}(t)\}_{t=1}^{\hat{t}}$, and the scheduling in slot $\tau, \tau > \hat{t}$ keeps unchanged. ■

Moreover, after the rescheduling in each iteration, with the updated subsets $\mathcal{S}(\bar{t})$ and $\mathcal{S}(\hat{t})$, we still have $s_t \geq c_t$ for $t < \hat{t}$. Therefore, the next time slot t with $s_t < c_t$ can only be greater than or equal to the current $\hat{t}$. Hence the assumption of Lemma 3 is still satisfied for the next iteration. Using induction, in each iteration of the rescheduling, a) and b) are always true.

Theorem 1 *The rescheduling procedure always finishes with a valid sensing policy with scheduling structure* $\{c_t\}_{t=1}^T$.

Proof: The proof of the feasibility of the rescheduling procedure includes three parts: First, we prove that in each iteration, we must have $t_{n-1} < \bar{t} < \hat{t} \leq t_n$ for some n. Second, with given $\bar{t}$ and $\hat{t}$, we can always find δ active sensors from $\mathcal{S}(\bar{t}) \backslash \mathcal{S}(\hat{t})$. Third, we prove that for any given $t_{n-1} < \hat{t} \leq t_n$, we can always find $c_{\hat{t}} - s_{\hat{t}}$ active nodes from time slots over $[t_{n-1}+1, \hat{t}-1]$.

The first part can then be proved through contradiction. According to Lemma 3, we always have $\bar{t} < \hat{t}$. Assume

$\bar{t} \le t_{n-1} < \hat{t} \le t_n$ for some n. Since $s_{\bar{t}} > c_{\bar{t}}$, we must have $\sum_{j=1}^{\bar{t}} s_j > \sum_{j=1}^{\bar{t}} c_j$. Therefore, $\sum_{j=1}^{t_{n-1}} s_j > \sum_{j=1}^{t_{n-1}} c_j = S(t_{n-1})$, where the last equality follows from Lemma 1. This implies that the energy causality constraint (7) is violated at t_{n-1}, which contradicts with the fact the energy causality constraint is always satisfied in each iteration. Thus, we must have $t_{n-1} < \bar{t} < \hat{t} \le t_n$.

To prove the second part, we note since $s_{\bar{t}} > c_{\bar{t}}$, $s_{\hat{t}} < c_{\hat{t}}$, and $t_{n-1} < \bar{t} < \hat{t} \le t_n$, applying Lemma 2, we have $s_{\bar{t}} > s_{\hat{t}}$. Therefore,

$$|\mathcal{S}(\bar{t}) \backslash \mathcal{S}(\hat{t})| \ge s_{\bar{t}} - s_{\hat{t}} \ge c_{\hat{t}} - s_{\hat{t}} \tag{11}$$

which ensures that we can always select δ active sensors from $\mathcal{S}(\bar{t}) \backslash \mathcal{S}(\hat{t})$.

To prove the last part, we let $\delta_t = s_t - c_t$. Then, $\delta_t \ge 0$ for $1 \le t < \hat{t}$, $\delta_{\hat{t}} < 0$, and

$$\sum_{t=1}^{\hat{t}} \delta_t = \sum_{t=1}^{\hat{t}} (s_t - c_t) = S(\hat{t}) - \sum_{t=1}^{\hat{t}} c_t \ge 0 \tag{12}$$

where the equality follows from Lemma 3, and the inequality follows from Lemma 1. Therefore, we can always remove $-\delta_{\hat{t}}$ nodes from $\{\mathcal{S}(t), t < \hat{t}\}$ to $\mathcal{S}(\hat{t})$, until we bring $s_{\hat{t}}$ to $c_{\hat{t}}$.
■

Theorem 2 *The obtained sensing scheduling with the structure* $\{c_t\}_{t=1}^T$ *determined by Algorithm 1 is optimal.*

IV. A Numerical Example

In this section, we use an numerical example to illustrate our scheduling algorithm under an offline setting. We consider a sensor network with 5 sensor nodes. The amount of energy harvested at each sensor node in slot $t-1$, $t \in [1, 10]$ is provided in the following table.

t	1	2	3	4	5	6	7	8	9	10
Node 1	4			2				1		
Node 2		5				2				
Node 3	1			3				7		
Node 4		1			5				2	
Node 5						7				
c_t	2	2	3	3	4	3	4	4	4	4

TABLE I: The energy harvesting profile for sensors over duration $[1, 10]$. The last line represents the number of active sensors in each slot obtained by Algorithm 1.

We then illustrate the procedure to obtain a feasible scheduling with the given scheduling structure $\{c_t\}_{t=1}^{10}$. The initial greedy scheduling is illustrated in Fig. 1(a), where we use a dot and a circle to represent the *active* and *idle* status of a node in a given time slot, respectively.

We then perform the rescheduling according to the procedure described in Sec. III-B, and obtain the final scheduling in Fig. 1(b). We note that a subset of sensor nodes change their status from *busy* to *idle* in certain time slots, and the saved energy is used in a time slot later. The final scheduling has exact c_t active sensors in slot t.

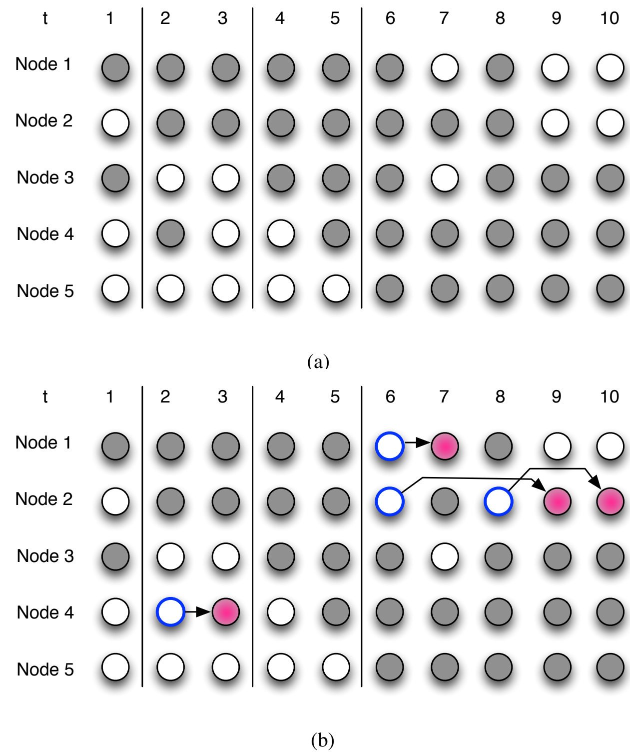

Fig. 1: Filled circles represent active sensors in each time slot under the initial scheduling and final scheduling in Fig. 1(a) and Fig. 1(b), respectively. Arrows connecting a blue circle and a red filled circle in Fig. 1(b) indicates the scheduling adjustments upon the initialization.

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