The Asymptotic Equivalence between Sensing Systems with Energy Harvesting and Conventional Energy Sources

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***Abstract*—In this paper, we seek answer to the question: can a wireless sensing system with energy harvesting power supplies perform as well as one with conventional power supplies? Due to the stochastic nature of the energy harvested from the ambient environment, uniform sampling employed by conventional sensing systems is usually infeasible for energy harvesting sensing systems. We propose a simple best-effort sensing scheme, which defines a set of equally-spaced candidate sensing instants. At a given candidate sensing instant, the sensor will perform sensing if there is sufficient energy available, and it will remain silent otherwise. It is analytically shown that the percentage of silent candidate sensing instants diminishes as time increases, if and only if the average energy harvesting rate is no less than the average energy consumption rate. The theoretical results are then used to guide the design of a practical sensing system that monitors a time-varying event. Both analysis and simulations show that the energy harvesting system with the best-effort sensing scheme can asymptotically achieve the same mean squared error (MSE) performance as one with uniform sensing and deterministic energy sources. Therefore, we provide a positive answer to the question from both theoretical and practical aspects.**

I. Introduction

Wireless sensing systems are usually expected to operate uninterruptedly and autonomously over years or decade under extremely stringent energy constraints. Such design objectives necessitate the development of systems powered by energy harvesting devices that can collect energy from the ambient environment. The amount of harvested energy can be modeled as a stochastic process. The stochastic nature of the energy sources is fundamentally different from the deterministic energy sources employed by conventional systems.

There have been growing interests in the development of energy harvesting communications and sensing systems by optimizing communication related metrics, such as transmission delay or throughput [13], [1], [11], [12]. Many of the works use an off-line deterministic scheduling method, which identifies the optimum transmission scheduling based on full knowledge of current and future energy arrivals. The off-line scheduling methods treats future energy arrival as a deterministic process even though the actual energy harvesting process is stochastic. Online scheduling methods address this problem by using only the statistics of energy arrivals. The off-line and online scheduling for systems with fading channels are discussed in [6], where the online scheduling problem is formulated as a stochastic dynamic programming problem with high complexity. Low complexity sub-optimum on-line algorithms are presented in [6] for fading channel and in [8] for interference channel. In both works, the performance of all on-line scheduling policies is strictly worse than that of the off-line scheduling.

Another branch of work focuses on sensing and signal processing related performance metrics, such as estimation mean squared error (MSE), and detection delay, etc. In [10], the estimation MSE of a sparse signal in an energy harvesting sensing system is minimized by using a random transmission scheme under the energy causality constraint. The sensing energy is assumed to be negligible in [10]. Optimum energy allocation schemes are discussed in [3] for the "quickest detection" of the changing point of an event monitored by energy harvesting sensor networks.

All of the works above show that the performance of energy harvesting systems is in general inferior to systems with conventional deterministic energy sources, even if optimal sensing policies are developed based on the stochastic properties of the energy sources.

Naturally we would ask the question: can an energy harvesting sensing system with stochastic energy sources perform as well as a conventional sensing system powered by deterministic energy sources? We seek the answer to this question from two aspects: from the theoretical aspect in terms of the stochastic properties of the available energy at a given time, and from the practical aspect in terms of the MSE performance of a sensing system. For a conventional sensing system with deterministic energy sources, uniform sampling is optimum over the duration in which the signal is wide sense stationary (WSS) [9], [7]. However, due to the randomness in the energy arrival process, uniform sampling might be infeasible in energy harvesting systems. We thus propose a simple best-effort sensing policy that defines a set of equally-spaced candidate sensing instants. At a given candidate sensing instant, the sensor will perform sensing only if there is sufficient energy to do so, and it will remain silent otherwise.

By analyzing the stochastic properties of the energy sources, we will show that the percentage of silent candidate sensing instants goes to zero as time goes to infinity, if and only if the

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average energy collection rate is no less than the average energy consumption rate. This means that the difference between the best-effort sensing policy and the uniform sensing policy diminishes as time evolves. The theoretical results indicate that the sensing behaviors of systems with stochastic energy sources have the potential to approach that of systems with deterministic energy sources. Guided by the theoretical results, we then develop optimum sensing and detection schemes for a practical energy harvesting sensing systems used to monitor a time-varying WSS event. It will be shown through both theoretical analysis and simulations that the MSE performance of the energy harvesting sensing system asymptotically approaches that of a conventional sensing system with uniform sensing as time goes to infinity. Simulation results show that the performance of the two systems converge with as few as 400 candidate sensing instants. Therefore, we demonstrate from both theoretical and practical aspects that there is an asymptotical equivalence between stochastic and deterministic energy sources.

II. Problem Formulation

Consider a sensor used to monitor a time-varying event modeled as a WSS random process $s(t)$, where t is the time variable. It is assumed that $s(t)$ is zero mean with auto-covariance function $r(t_1-t_2)=\mathbb{E}[s(t_1)s(t_2)]=\rho^{|t_1-t_2|}$, where $\mathbb{E}$ is the mathematical expectation operator, and $0\leq\rho\leq 1$ is the power-law coefficient.. The sensor is powered by an energy harvesting device, which harvests energy from the ambient environment. The harvested energy can be modeled as a random process, and it is used by the sensor for sensing operations.

The sensor attempts to reconstruct the continuous-time time-varying random event by using noise-distorted discrete-time observations of the random process. A sensing policy is defined as a sequence of time instants $\{t_n\}_n$, where t_n is the time instant at which the sensor collects a sample of the random process. The sample collected by the sensor at time t_n is

$$y(t_n)=\sqrt{E_s}s(t_n)+z(t_n) \tag{1}$$

where E_s is the energy allocated for one sensing sample, and $z(t_n)$ is the sensing and/or channel noise with a zero-mean and the auto-covariance function $\mathbb{E}\left[z(t_1)z(t_2)\right]=\sigma_z^2\delta(t_1-t_2)$. It should be noted that the noise component is not necessarily Gaussian distributed.

The sensing system attempts to reconstruct the time-varying random field, $s(t)$, by using the sequence of the discrete-time samples, $\{y(t_n)\}_n$. If the sensor is powered by a conventional power supply, then uniform sampling with $t_n=nT_s$ is employed in [7], [9] to achieve the optimum sensing performance due to the homogenous nature of the random field. However, uniform sampling might be infeasible for energy harvesting sensing systems given that there might not be sufficient energy to perform sensing operations at certain time periods.

III. Asymptotic Equivalence between Stochastic and Deterministic Energy Sources

In this section, we first propose a best-effort sensing policy for systems with stochastic energy sources. We then study the asymptotic behaviors of the best-effort sensing policy as time goes to infinity. The analytical results demonstrate the asymptotic equivalence between stochastic energy sources and deterministic energy sources.

Due to the random nature of the energy harvesting process, the harvested energy can be modeled as a random process. We model the stochastic energy sources as follows: if we divide the time axis into small intervals with length T_0, then the energy collected in each interval can be modeled as independently and identically distributed (i.i.d.) random variables E with mean E_0. In addition, $\sum_{n=1}^{\infty}P(E>n\epsilon)<\infty$ for any $\epsilon>0$. Such a model is general enough to incorporate many other existing stochastic energy models, such as the Poisson energy source [6] or the Bernoulli energy source [3], as special cases.

The harvested energy is stored in an energy storage device, such as rechargeable batteries or super capacitors. Denote the amount of energy available in the energy storage device at time t as $Q(t)\geq 0$. Since the harvested energy is usually very small compared to the capacity of the energy storage device, it is assumed that the energy queue has unlimited capacity. The energy consumption must follow the energy causality constraint, that is, at any time instant, the total amount of harvested energy must be no less than the total amount of consumed energy.

Definition 1 (best-effort Sensing Policy): Define a set of candidate sensing instants as $\mathcal{K}=\{k_n|k_n=nT_s,n=1,2,\cdots\}$. A sensor performs one sensing operation with energy E_s at time t if and only if: 1) $t\in\mathcal{K}$, and 2) $Q(t)\geq E_s$.

In the best-effort sensing policy, the sensor tries to perform sensing operations at uniform sensing intervals whenever allowed by the energy constraint, but remains silence if $Q(nT_s)<E_s$. Denote the information collected at each candidate sensing instant as a sensing symbol, which could be either a silent symbol when $Q(nT_s)<E_s$ or an active symbol when $Q(nT_s)\geq E_s$. With such a sensing mechanism and the stochastic energy source, there might be K silent symbols in the first $N\geq K$ sensing instants $T_s,2T_s,\cdots,NT_s$. The number of silent symbols is a random variable.

Theorem 1: Consider an energy harvesting sensing system employing the best-effort sensing policy described in Definition 1. Assume the amounts of energy collected in each T_s period are i.i.d random variables E with mean E_c, and $\sum_{n=1}^{\infty}P[E>n\epsilon]<\infty$ for any $\epsilon>0$. Define $K=\sum_{k=1}^{N}\mathbf{1}_{Q(kT_s)<E_s}$ as the total number of silent symbols in the first N symbol periods, where the indicator function $\mathbf{1}_{\mathcal{E}}=1$ if the event $\mathcal{E}$ is true and 0 otherwise.

If $E_c\geq E_s$, then

$$\lim_{N\to\infty}\frac{K}{N}=0,\quad \text{a.s.} \tag{2}$$

Specifically, if $E_c>E_s$, then for almost every energy harvesting sample path, there exists $\bar{K}<\infty$ such that $K\leq\bar{K}<\infty$

as $N \to \infty$.

Conversely, If $E_c < E_s$, then

$$\lim_{N\to\infty} \frac{K}{N} > 1 - \frac{E_c}{E_s}, \quad \text{a.s.} \tag{3}$$

Proof: Divide the time axis into frames, each of duration LT_s. The m-th frame thus has L candidate sensing instants, and $k_m \le L$ of them are assumed to be silent. Assume the total amount of energy collected in the m-th frame is E_m, which is random. The amount of energy consumed in the l-th frame can be calculated as $(L-k_m)E_s$. Denote $D_m = E_m - (L-k_m)E_s$ as the difference between the energy harvested and consumed in the m-th frame. It should be noted that D_m could be either positive or negative. The total amount of energy available in the energy queue at the end of the m-th frame is $Q(mLT_s) = \sum_{m=1}^{M} D_m$. It should be noted that $\sum_{m=1}^{M} D_m \ge 0$ due to the energy causality constraint.

With the best-effort sensing policy, the number of silent symbols in the M-th frame must satisfy

$$k_{_M} \le \max\left\{0, L - \frac{1}{E_s}\sum_{m=1}^{M-1} D_m\right\}, \tag{4}$$

because the energy available at the end of the $(M-1)$-th frame can be used for the sensing of up to $\frac{1}{E_s}\sum_{m=1}^{M-1} D_m$ symbols in the M-th frame.

1) Case 1: $E_s > E_c$. With the energy causality constraint, we have $\sum_{m=1}^{M} D_m \ge 0$, or

$$\sum_{m=1}^{M} E_m - MLE_s + E_s \sum_{m=1}^{M} k_m \ge 0 \tag{5}$$

Divide both sides of (5) by MLE_s, and let $M \to \infty$,

$$\lim_{M\to\infty} \frac{\sum_{m=1}^{M} k_m}{ML} \ge 1 - \frac{1}{E_s} \lim_{M\to\infty} \frac{\sum_{m=1}^{M} E_m}{ML} \tag{6}$$

Based on the strong law of large numbers,

$$\lim_{M\to\infty} \frac{\sum_{m=1}^{M} E_m}{ML} = E_c, \quad \text{a.s.} \tag{7}$$

In addition, let $N = LM$, then $K = \sum_{m=1}^{M} k_m$. Thus (3) can be obtained from (6) and (7).

2) Case 2: $E_c = E_s$. Index the frames with at least one silent symbol as $M_1, M_2, \cdots, M_i, \cdots$, i.e.,

$$0 < k_{_{M_i}} \le L - \frac{\sum_{m=1}^{M_i-1} D_m}{E_s}. \tag{8}$$

If M_i is upperbounded, that is, there exists $\bar{M}$ such that $k_m = 0$ for all $m > \bar{M}$, then K is finite and (2) is true. On the other hand, if M_i is unbounded, then $\lim_{i\to\infty} M_i = \infty$. We have

$$\sum_{m=1}^{M_i} D_m = \sum_{m=1}^{M_i-1} D_m + E_{_{M_i}} - LE_s + k_{_{M_i}} E_s \le E_{_{M_i}}, \tag{9}$$

where the last inequality is based on (8). Dividing both sides of (9) by LM_i and letting $i \to \infty$ (so as M_i), we have

$$\lim_{M_i\to\infty} \frac{\sum_{m=1}^{M_i} E_m}{LM_i} - E_s + \lim_{M_i\to\infty} \frac{\sum_{m=1}^{M_i} k_m}{LM_i} \le \lim_{M_i\to\infty} \frac{E_{M_i}}{LM_i} \tag{10}$$

Based on the assumption that $\sum_{n=1}^{\infty} P[E > n\epsilon] < \infty$ for any $\epsilon > 0$, and Borel-Cantelli lemma [5], we have

$$\lim_{M_i\to\infty} \frac{E_{M_i}}{LM_i} = 0, \quad \text{a.s.}$$

Combining (10) with (7) yields

$$\lim_{N\to\infty} \frac{K}{N} \le 0, \quad \text{a.s.} \tag{11}$$

Since $\frac{K}{N} \ge 0$, (2) is true.

3) Case 3: $E_c > E_s$. Proof by contradiction. Assume $\lim_{i\to\infty} M_i = \infty$. When $E_c > E_s$, from (7) and (10), we have

$$\lim_{N\to\infty} \frac{K}{N} \le -(E_c - E_s) < 0, \quad \text{a.s.} \tag{12}$$

This contradicts with the fact that $\frac{K}{N} \ge 0$, thus the assumption $\lim_{i\to\infty} M_i = \infty$ cannot be true when $E_c > E_s$. This means that M_i is finite, thus K is finite as $N \to \infty$. ■

The results in Theorem 1 state that, there is an asymptotic equivalence between stochastic energy source and deterministic energy source as time goes to infinity, if and only if the average harvested energy in one sample period is no less than the energy required for one sensing operation. This means that the performance of the best-effort sensing policy with stochastic energy sources has the potential to match that of the uniform sensing policy with deterministic energy sources as time becomes large enough and $E_c \ge E_s$. The results hold for a quite general category of energy harvesting processes.

IV. Asymptotically Optimum Sensing with the best-effort Sensing Policy

This section studies the optimum design and performance analysis of sensing systems employing the newly proposed best-effort sensing policy.

A. *Best-effort Sensing with Stochastic Energy Sources*

Based on the best-effort sensing policy, the samples collected during the first N candidate sensing instants can be expressed by

$$\boldsymbol{\eta} = \sqrt{E_s}\mathbf{x} + \mathbf{z} \tag{13}$$

where $\boldsymbol{\eta} = [\eta_1, \cdots, \eta_N]^T \in \mathcal{R}^N$, $\mathbf{x} = [x_1, \cdots, x_n]^T$ with $x_n = s(nT_s)$ if $Q(nT_s) \ge E_s$ and $x_n = 0$ otherwise, and $\mathbf{z} = [z(T_s), \cdots, z(NT_s)]^T$. With the best-effort sensing policy, K out of the N elements in $\boldsymbol{\eta}$ contain only noise components.

The system attempts to reconstruct the time-varying random event $s(t)$ by using the observations, $\boldsymbol{\eta}$. Since we are interested in the reconstruction fidelity of a continuous-time random event, the worst case scenario will be considered by estimating $\{s\left(nT_s + \frac{1}{2}T_s\right)\}_n$, the sequence of points located in the middle between two candidate sensing instants. Define the data vector to be estimated as $\mathbf{d} = \left[s\left(\frac{1}{2}T_s\right), s\left(\frac{3}{2}T_s\right), \cdots, s\left(NT_s - \frac{1}{2}T_s\right)\right]^T$. It should be noted that $s\left(nT_s + \frac{1}{2}T_s\right)$ will be estimated even if $x_n = 0$ and/or $x_{n+1} = 0$.

The linear minimum mean squared error (MMSE) estimation of $\mathbf{d}$ based on $\boldsymbol{\eta}$ is

$$\hat{\mathbf{d}} = \sqrt{E_s}\mathbf{R}_{dx}\left[E_s\mathbf{R}_{xx} + \mathbf{R}_{zz}\right]^{-1}\boldsymbol{\eta}, \quad (14)$$

where $\mathbf{R}_{dx} = \mathbb{E}(\mathbf{d}\mathbf{x}^T)$, $\mathbf{R}_{xx} = \mathbb{E}(\mathbf{x}\mathbf{x}^T)$, and $\mathbf{R}_{zz} = \sigma_z^2\mathbf{I}_N$ with $\mathbf{I}_N$ being a size $N \times N$ identity matrix. If we assume the indices of the sampling instants with $Q(nT_s) < E_s$ being $i_1, i_2, \cdots, i_K$, then the i_k-th column of $\mathbf{R}_{dx}$ is an all-zero column, for $k = 1, \cdots, K$. Similarly, the i_k-th row and the i_k-th column of $\mathbf{R}_{xx}$ are all-zero vectors, for $k = 1, \cdots, K$.

The covariance matrix of the error vector, $\boldsymbol{\epsilon} = \mathbf{d} - \hat{\mathbf{d}}$, can be written as

$$\mathbf{R}_{\epsilon\epsilon} = \mathbf{R}_{dd} - \mathbf{R}_{dx}\left[\mathbf{R}_{xx} + \frac{\sigma_z^2}{E_s}\mathbf{I}_N\right]^{-1}\mathbf{R}_{xd}. \quad (15)$$

Systems with deterministic energy sources can be considered as a special case of the MMSE receiver described in (13) and (14). With a deterministic energy source, there will be no silent symbols, thus the system equation can be expressed as,

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{z} \quad (16)$$

where $\mathbf{y} = [y(T_s), \cdots, y(NT_s)]^T$ and $\mathbf{s} = [s(T_s), \cdots, s(NT_s)]^T$ are the received signal vector and data vector, respectively. Denote the MMSE estimation of $\mathbf{d}$ from $\mathbf{y}$ as $\tilde{\mathbf{d}} = \mathbf{W}_y^T\mathbf{y}$, where $\mathbf{W}_y^T$ is the linear MMSE matrix. Similar to (15), the covariance matrix of the error vector, $\mathbf{e} = \mathbf{d} - \tilde{\mathbf{d}}$, can be written as

$$\mathbf{R}_{ee} = \mathbf{R}_{dd} - \mathbf{R}_{ds}\left[\mathbf{R}_{ss} + \frac{\sigma_z^2}{E_s}\mathbf{I}_N\right]^{-1}\mathbf{R}_{sd} \quad (17)$$

where $\mathbf{R}_{ds} = \mathbb{E}(\mathbf{d}\mathbf{s}^T)$ and $\mathbf{R}_{ss} = \mathbb{E}(\mathbf{s}\mathbf{s}^H)$ are Toeplitz matrices. The cross-covariance matrix $\mathbf{R}_{ds}$ is a Toeplitz matrix with the first row being $\left[r\left(\frac{1}{2}T_s\right), r\left(\frac{3}{2}T_s\right), \cdots, r\left(NT_s - \frac{1}{2}T_s\right)\right]$, and the first column $\left[r\left(\frac{1}{2}T_s\right), r\left(\frac{1}{2}T_s\right), r\left(\frac{3}{2}T_s\right), \cdots, r\left(NT_s - \frac{3}{2}T_s\right)\right]^T$. The covariance matrix $\mathbf{R}_{ss}$ is a symmetric Toeplitz matrix with the first row being $[r(0), r(T_s), \cdots, r(NT_s - T_s)]$.

The error covariance matrices for systems with stochastic and deterministic energy sources are given in (15) and (17), respectively. The average MSE for systems with stochastic and deterministic energy sources can then be calculated, respectively, as $\sigma_{\epsilon,N}^2 = \frac{1}{N}\text{trace}\left(\mathbf{R}_{\epsilon\epsilon}\right)$, and $\sigma_{e,N}^2 = \frac{1}{N}\text{trace}\left(\mathbf{R}_{ee}\right)$.

Intuitively, $\sigma_{\epsilon,N}^2 \geq \sigma_{e,N}^2$ since $\mathbf{y}$ contains more information than $\boldsymbol{\eta}$. However, due to asymptotic equivalence between the stochastic and deterministic energy sources as presented in Theorem 1, we will show in the next subsection that the proposed best-effort sensing policy can asymptotically achieve the same performance as the uniform sensing policy as $N \to \infty$.

B. Asymptotic Achievability of the MSE Lower Bound

The asymptotic equivalence between the two sensing schemes relies on the asymptotic equivalence between sequences of matrices.

Lemma 1: Consider two sequences of matrices, $\{\mathbf{A}_N\}_N$ and $\{\mathbf{B}_N\}_N$. The squares of the elements on any row or column of the two matrices are absolutely summable as $N \to \infty$. If the two matrices differ in K_1 rows and K_2 columns, and $\lim_{N\to\infty}\frac{K_1+K_2}{N} = 0$, then $\mathbf{A}_N$ and $\mathbf{B}_N$ are asymptotically equivalent, which is denoted as $\mathbf{A}_N \sim \mathbf{B}_N$.

Proof: Denote the (m,n)-th elements of $\mathbf{A}_N$ and $\mathbf{B}_N$ as a_{mn} and b_{mn}, respectively. Since the squares of the elements on each row or column of the two matrices are absolutely summable, the matrices are bounded in strong norm. Assume the two matrices differ in rows $r_1, \cdots, r_{K_1}$ and columns $c_1, \cdots, c_{K_2}$, then

$$|\mathbf{A}_N - \mathbf{B}_N|^2 \leq \frac{1}{N}\left[\sum_{k=1}^{K_1}\sum_{n=1}^{N}|a_{r_k n} - b_{r_k n}|^2 + \sum_{m=1}^{M}\sum_{k=1}^{K_2}|a_{mc_k} - b_{mc_k}|^2\right] \quad (18)$$

Since the squares of the elements on any row or column are absolutely summable, then there exists $C > 0$ such that

$$\sum_{n=1}^{\infty}|a_{r_k n} - b_{r_k n}|^2 \leq \sum_{n=1}^{\infty}|a_{r_k n}|^2 + \sum_{n=1}^{\infty}|b_{r_k n}|^2 < 2C$$
$$\sum_{m=1}^{\infty}|a_{mc_k} - b_{mc_k}|^2 \leq \sum_{m=1}^{\infty}|a_{mc_k}|^2 + \sum_{m=1}^{\infty}|b_{mc_k}|^2 < 2C$$

Thus

$$\lim_{N\to\infty}|\mathbf{A}_N - \mathbf{B}_N| < \sqrt{4C}\lim_{N\to\infty}\sqrt{\frac{K}{N}} \quad (19)$$

Since $\lim_{N\to\infty}\frac{K}{N} = 0$, it is straightforward that $\lim_{N\to\infty}|\mathbf{A}_N - \mathbf{B}_N| = 0$. ■

From (15) and (17), σ_ϵ^2 and $\mathbf{R}_{\epsilon\epsilon}$ depend on $\mathbf{R}_{dx}$ and $\mathbf{R}_{xx}$, while σ_e^2 and $\mathbf{R}_{ee}$ depend on $\mathbf{R}_{ds}$ and $\mathbf{R}_{ss}$. Since $\lim_{N\to\infty}\frac{K}{N} = 0$ a.s. when $E_c \geq E_s$, the following result follows immediately from Lemma 1.

Lemma 2: If $E_c \geq E_s$ in the best-effort sensing policy, then $\mathbf{R}_{dx} \sim \mathbf{R}_{ds}$, and $\mathbf{R}_{xx} \sim \mathbf{R}_{ss}$.

Now we are ready to present the second main result of this paper in the following theorem, which states the asymptotic equivalence between systems with stochastic and deterministic energy sources in terms of MSE performance.

Theorem 2: Consider two sensing systems, one with the best-effort sensing policy and a stochastic energy source as described in (13), and one with the uniform sensing policy and a deterministic energy source as described in (16). If $E_c \geq E_s$, then

$$\lim_{N\to\infty}\sigma_{\epsilon,N}^2 = \lim_{N\to\infty}\sigma_{e,N}^2. \quad (20)$$

Proof: Based on [2, Lemma 2], the Toeplitz matrix $\mathbf{R}_{ds}$ is asymptotically equivalent to a circulant matrix, $\mathbf{C}_{ds} = \mathbf{U}_N^H\mathbf{D}_{ds}\mathbf{U}_N$, where $\mathbf{U}_N$ is the unitary discrete Fourier transform (DFT) matrix with the (m,n)-th element being $(\mathbf{U}_N)_{m,n} = \frac{1}{\sqrt{N}}\exp\left[-j2\pi\frac{(m-1)(n-1)}{N}\right]$, and $\mathbf{D}_{ds}$ is a diagonal matrix with its k-th diagonal element being $(\mathbf{D}_{ds})_{k,k} = \Lambda_{ds}\left(\frac{k-1}{N}\right)$, where Λ_{ds} is the discrete-time Fourier transform (DTFT) of the elements on the first column and row of $\mathbf{R}_{ds}$.

Similarly, the symmetric Toeplitz matrix $\mathbf{R}_{ss}$ is asymptotically equivalent to a circulant matrix, $\mathbf{C}_{ss} = \mathbf{U}_N^H \mathbf{D}_{ss} \mathbf{U}_N$, where $\mathbf{D}_{ss}$ is a diagonal matrix with its k-th diagonal element being $(\mathbf{D}_{ss}) = \Lambda_{ss}\left(\frac{k-1}{N}\right)$, where Λ_{ss} is the DTFT of the elements on the first column and row of $\mathbf{R}_{ss}$.

When $E_c \geq E_s$, we have $\lim_{N\to\infty} \frac{K}{N} = 0$ from Theorem 1. Based on Lemma 2 and [4, Theorem 2.1], we have

$$\mathbf{R}_{dx} \sim \mathbf{R}_{ds} \sim \mathbf{C}_{ss} \quad (21)$$

$$\mathbf{R}_{xx} \sim \mathbf{R}_{ss} \sim \mathbf{C}_{ss} \quad (22)$$

Therefore, based on [4, Theorem 2.1], we have $\mathbf{R}_{\epsilon\epsilon} \sim \mathbf{R}_{ee} \sim \mathbf{C}_{ee}$, where $\mathbf{C}_{ee}$ is a circulant matrix defined as

$$\mathbf{C}_{ee} = \mathbf{C}_{ss} - \mathbf{C}_{ds}\left[\mathbf{C}_{ss} + \frac{\sigma_z^2}{E_s}\mathbf{I}_N\right]^{-1}\mathbf{C}_{ds}^H \quad (23)$$

The circulant matrix $\mathbf{C}_{ee}$ can be expressed as $\mathbf{C}_{ee} = \mathbf{U}_N^H \mathbf{D}_{ee} \mathbf{U}_N$, where $\mathbf{D}_{ee}$ is a diagonal matrix defined as

$$\mathbf{D}_{ee} = \mathbf{D}_{ss} - \mathbf{D}_{ds}\left[\mathbf{D}_{ss} + \frac{\sigma_z^2}{E_s}\mathbf{I}_N\right]^{-1}\mathbf{D}_{ds}^H. \quad (24)$$

Therefore, both error covariance matrices are asymptotically equivalent to the same circulant matrix. Based on Szego's Theorem [2], when $N \to \infty$, we have

$$\lim_{N\to\infty} \sigma_{\epsilon,N}^2 = \lim_{N\to\infty} \sigma_{e,N}^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\Lambda_{ss}(f) - \frac{|\Lambda_{ds}(f)|^2}{\Lambda_{ss}(f) + \frac{\sigma_z^2}{E_s}}\right] df.$$

■

Theorem 1 provides the theoretical foundation on the asymptotic equivalence between stochastic and deterministic energy sources, and Theorem 2 demonstrates such equivalence can be achieved in practical systems.

V. Numerical and Simulation Results

Numerical and simulation results are presented in this section to demonstrate the asymptotic equivalence between systems with stochastic and deterministic energy sources.

Fig. 1 shows the value of $\frac{K}{N}$ as a function of N for systems with various stochastic energy sources, where K is the number of silent sensing symbols and N is the total number of candidate sensing symbols. All systems employ the best-effort sensing policy proposed in this paper. The convergence behaviors of $\frac{K}{N}$ are demonstrated for $E_c = 0.9E_s$, $E_c = E_s$, and $E_c = 1.1E_s$, respectively. The simulation results are obtained by averaging over 100 independent runs for each configuration. The exponential energy source is modeled as $E = |V(t)|^2$, where $V(t)$ is a zero-mean symmetric complex Gaussian random process with covariance function $\mathbb{E}[V(t+\tau)V^*(t)] = \sigma_v^2\delta(\tau)$. The energy collected in an interval T_s thus follows an exponential distribution with mean $E_s = \sigma_v^2 T_s$. All three energy sources have similar convergence behaviors as N increases. When $E_c \geq E_s$ and $N > 100$, the value of $\log\frac{K}{N}$ decreases almost linearly with respect to $\log N$, and the absolute slope increases as $\frac{E_c}{E_s}$ increases. The simulation results conform to Theorem 1, which states that $\lim_{N\to\infty}\frac{K}{N} = 0$ when $E_c \geq E_s$. On the other hand, when $E_c = 0.9E_s$, $\frac{K}{N}$ tends to a constant value 0.1, as $N \to \infty$. The constant value is the same as the lower bound, $1 - \frac{E_c}{E_s}$, predicted by Theorem 1. The results in Fig. 1 demonstrate through simulations that the percentage of silent sensing symbols diminish as time goes to infinity as long as $E_c \geq E_s$. The actual distributions of the energy sources have no impact on the convergence behavior.

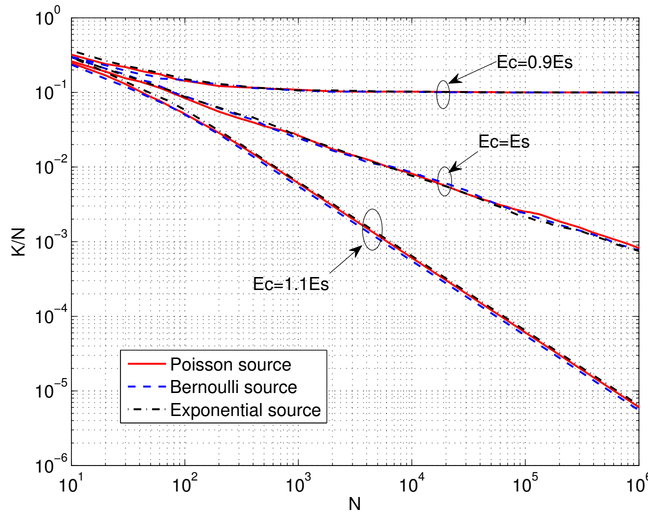

Fig. 1. K/N for various energy sources.

Fig. 2 compares the MSE performance of systems with both stochastic and deterministic energy sources. The best-effort sensing policy is employed by systems with stochastic energy sources, and uniform sensing is employed by systems with deterministic energy sources. The stochastic energy sources are the exponential sources with mean $E_c = E_s$. All systems have the same average power $P = \frac{E_s}{T_s}$ with the normalized signal-to-noise ratio (SNR) $\gamma_0 = \frac{P}{\sigma_z^2}$ being 5 dB. The power-law coefficient is $\rho = 0.9$. As expected, the MSE performance of systems with deterministic energy sources is consistently better than that of systems with stochastic energy sources. However, the performance gap narrows as N increases. When $N = 400$, the MSE performance of the two systems are almost identical, and they coincide with the asymptotic MSE obtained with $N \to \infty$. Therefore, when N is sufficiently large, the best-effort sensing policy can achieve a performance that is almost the same as the uniform sensing policy. Thus the results in Fig. 2 demonstrate the asymptotic equivalence between stochastic and deterministic energy sources in a practical system.

In addition, it can be seen from Fig. 2 that the MSE is convex when N is small ($N = 10$ or 50), and it becomes a monotonically decreasing function when N is large ($N = 400$). Under a fixed power, a larger sampling rate means less energy per sample, which might degrade the system performance. On the other hand, a larger sampling rate means a stronger correlation between two adjacent samples, which contributes positively to the MSE performance. Therefore changing the sampling rate results in different tradeoffs between energy per

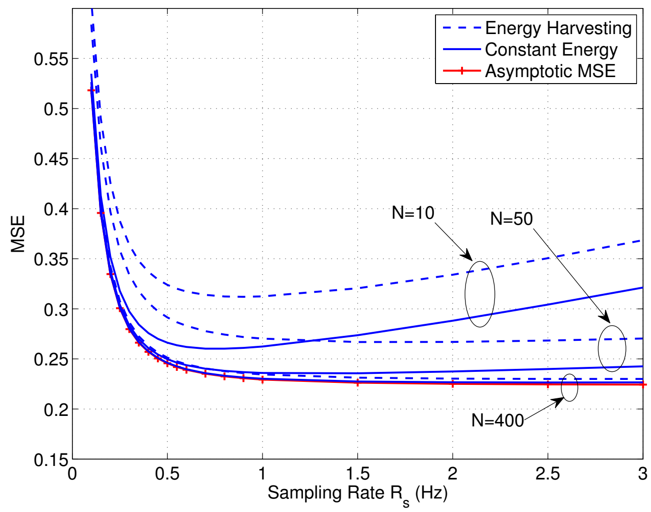

Fig. 2. MSE as a function of the sampling frequency ($\rho = 0.9$).

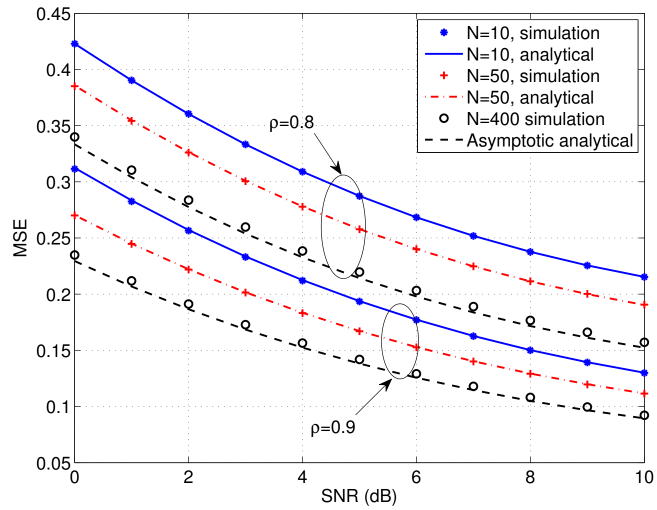

Fig. 3. MSE as a function of the SNR for systems with stochastic energy sources ($R_s = 1$ Hz).

sample and sample correlations. When N is large enough, increasing the sampling rate beyond a certain threshold (e.g. $R_s = 1$ Hz when $N = 400$) has negligible impact on the MSE performance.

Fig. 3 shows the simulation and analytical MSE of systems with stochastic energy sources as functions of the SNR. The sampling rate is 1 Hz. The analytical results with finite N are calculated from (15). The simulation results are obtained through Monte Carlo simulations, where the data are generated as a zero-mean Gaussian random process with the covariance function satisfying the power-law relationship. Each point on the curve is averaged over 1,000 independent simulation runs. The simulation results match the analytical results very well. As expected, the MSE decreases as ρ or N increases. In addition, the MSE with $N = 400$ is almost the same as the asymptotic MSE obtained with $N \to \infty$.

VI. Conclusions

The asymptotic equivalence between stochastic and deterministic energy sources have been demonstrated through both theoretical analysis and practical examples. To account for the stochastic nature of energy harvested from the ambient environment, a best-effort sensing policy has been proposed for energy harvesting sensing systems. It has been shown that the difference between the best-effort sensing scheme and the ideal uniform sensing scheme diminishes as time goes to infinity, if and only if the average energy collection rate is no less than the average energy consumption rate, regardless of the actual distribution of the stochastic energy source. The asymptotic equivalence has been used for the development of an optimum energy harvesting sensing system. It has been shown through both theoretical analysis and simulation results that systems with the best-effort sensing scheme and stochastic energy sources can achieve almost the same MSE performance as systems with uniform sensing and deterministic energy sources when the number of samples is greater than 400.

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