Optimal Packet Scheduling in a Multiple Access Channel with Rechargeable Nodes

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***Abstract*—In this paper, we investigate the optimal packet scheduling problem in a two-user multiple access communication system, where the transmitters are able to harvest energy from the nature. Under a deterministic system setting, we assume that the energy harvesting times and harvested energy amounts are known before the transmission starts. For the packet arrivals, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts. Our goal is to minimize the time by which all packets from both users are delivered to the destination through controlling the transmission powers and transmission rates of both users. We first develop a *generalized iterative backward waterfilling* algorithm to characterize the maximum departure region of the transmitters for any given deadline T. Then, based on the sequence of maximum departure regions at energy arrival epochs, we decompose the transmission completion time minimization problem into a convex optimization problem and solve it efficiently.**

I. INTRODUCTION

Efficient energy management is crucial for wireless communication systems, as it increases the throughput and improves the delay performance. Energy efficient scheduling policies have been well investigated in traditional battery powered (unrechargeable) systems [1]–[5]. On the other hand, there exist systems where the transmitters are able to harvest energy from the nature. Such energy harvesting abilities make sustainable and environmentally friendly deployment of communication systems possible. This renewable energy supply feature also necessitates a completely different approach to energy management.

In this work, we consider a multi-user rechargeable wireless communication system, where data packets as well as the harvested energy arrive at the transmitters as random processes in time. As shown in Fig. 1, we consider a two-user multiple access channel, where each transmitter node has two queues. The data queue stores the data arrivals, while the energy queue stores the energy harvested from the environment. Our objective is to adaptively change the transmission rate and power according to the instantaneous data and energy queue sizes, such that the transmission completion time is minimized.

In general, the arrival processes for the data and the harvested energy can be formulated as stochastic processes,

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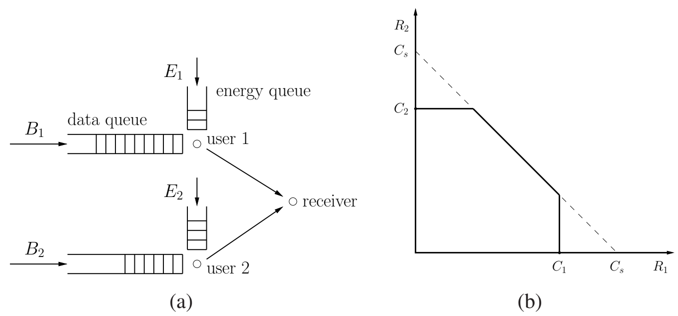

Fig. 1. (a) An energy harvesting multiple access channel model with energy and data queues, and (b) the capacity region of the additive white Gaussian noise multiple access channel.

and the problem requires an *on-line* solution that adapts transmission power and rate in *real-time*. Since this seems to be an intractable problem for now, we simplify the problem by assuming that the data packets and energy will arrive in a deterministic fashion, and we aim to develop an *off-line* solution instead. In this paper, we consider the scenario where packets have already arrived before the transmissions start. Specifically, we consider two nodes as shown in Fig. 2. For the traffic load, we assume that there are a total of B_1 bits and B_2 bits available at the first and second transmitter, respectively, at time $t = 0$. We assume that energy arrives (is harvested) at points in time marked with $\circ$. In Fig. 2, E_{1k} denotes the amount of energy harvested for the first user at time s_k. Similarly, E_{2k} denotes the amount of energy harvested for the second user at time s_k. If there is no energy arrival at one of the nodes, we simply let the corresponding amount be zero, which are denoted by the dotted arrows in Fig. 2. Our goal then is to develop methods of transmission to minimize the time, T, by which all of the data packets from both of the nodes are delivered to the destination.

The optimal packet scheduling problem in a single-user energy harvesting communication system is investigated in [6], [7]. In [6], [7], we prove that the optimal scheduling policy has a "majorization" structure, in that, the transmit power is kept constant between energy harvests, the sequence of transmit powers increases monotonically, and only changes at some of the energy harvesting instances; when the transmit power changes, the energy constraint is tight, i.e., the total consumed energy equals the total harvested energy. In [6], [7], we develop an algorithm to obtain the optimal off-line scheduling policy based on these properties. Reference [8] extends [6], [7] to the case where rechargeable batteries have finite sizes. We extend [6], [7] in [9] to a fading channel.

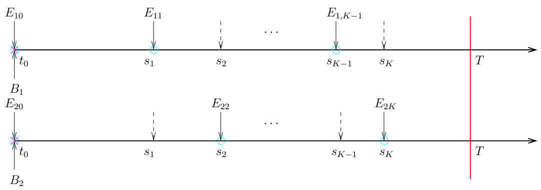

Fig. 2. System model with all packets available at the beginning. Energies arrive at points denoted by $\circ$.

In the two-user multiple access channel setting studied in this paper, the scheduling problem is significantly more complicated. This is because the two users interfere with each other, and we need to select the transmission powers for both users as well as the rates from the resulting rate region, to solve the problem. In addition, because the traffic load and the harvested energy for both users may not be well-balanced, the final transmission durations for the two users may not be the same, further complicating the problem.

We first investigate a problem which is "dual" to the transmission completion time minimization problem. In this "dual" problem, we aim to characterize the maximum number of bits both users can transmit for any given time T. These two problems are "dual" to each in the sense that, if (B_1, B_2) lies on the boundary of the maximum departure region for time T^*, then, T^* must be the solution to the transmission completion time minimization problem with initial number of bits (B_1, B_2). We propose a *generalized iterative backward waterfilling* algorithm to achieve the boundary points of the maximum departure region for any given time T. Then, based on the solution of this "dual" problem, we go back to the transmission completion time minimization problem, simplify it into standard convex optimization problems, and solve it efficiently.

II. System Model and Problem Formulation

The system model is as shown in Figs. 1 and 2. As shown in Fig. 1, each user has a data queue and an energy queue. The physical layer is modeled as an additive white Gaussian noise channel, where the received signal is

$$Y = X_1 + X_2 + Z \tag{1}$$

where X_i is the signal of user i, and Z is a Gaussian noise with zero-mean and unit-variance. The capacity region for this two-user multiple access channel is [10]

$$R_1 \leq f(P_1) \tag{2}$$
$$R_2 \leq f(P_2) \tag{3}$$
$$R_1 + R_2 \leq f(P_1 + P_2) \tag{4}$$

where $f(p) = \frac{1}{2}\log(1+p)$. We denote the region defined by these inequalities above as $\mathcal{C}(P_1, P_2)$. This region is shown on the right figure in Fig. 1.

As shown in Fig. 2, user i has B_i bits to transmit which are available at transmitter i at time $t = 0$. Energy is harvested at times s_k with amounts E_{ik} at transmitter i. Our goal is to solve for the transmit power sequence, the rate sequence, and the corresponding duration sequence that minimize the time T by which all of the bits are delivered to the destination.

Let us denote the transmit power for the first and second user at time t as $p_1(t)$ and $p_2(t)$, respectively. Then, the transmission rate pair $(r_1(t), r_2(t))$ must be within the capacity region defined by $p_1(t)$ and $p_2(t)$, i.e., $\mathcal{C}(p_1, p_2)(t)$. For user i, $i = 1, 2$, the energy consumed up to time t, denoted as $E_i(t)$, and the total number of bits departed up to time t, denoted as $B_i(t)$, can be written as:

$$E_i(t) = \int_0^t p_i(\tau)d\tau, \quad B_i(t) = \int_0^t r_i(\tau)d\tau, \quad i = 1, 2 \tag{5}$$

Here r_i and powers p_i are related through the f function as shown in (2)-(4). Then, the transmission completion time minimization problem can be formulated as:

$$\begin{aligned} \min_{p_1, p_2, r_1, r_2} \quad & T \\ \text{s.t.} \quad & E_1(t) \leq \sum_{n: s_n < t} E_{1n}, \quad 0 \leq t \leq T \\ & E_2(t) \leq \sum_{n: s_n < t} E_{2n}, \quad 0 \leq t \leq T \\ & B_1(T) \geq B_1, \quad B_2(T) \geq B_2 \\ & (r_1, r_2)(t) \in \mathcal{C}(p_1, p_2)(t), \; 0 \leq t \leq T \end{aligned} \tag{6}$$

III. Characterizing $\mathcal{D}(T)$: Largest (B_1, B_2) Region for a Given Deadline T

We define the maximum departure region as follows.

Definition 1 *For any fixed transmission duration T, the maximum departure region, denoted as $\mathcal{D}(T)$, is the union of (B_1, B_2) under any feasible power and rate allocation policy over the duration $[0, T)$.*

We call any policy which achieves the boundary of $\mathcal{D}(T)$ to be optimal.

Lemma 1 *Under the optimal policy, the transmission power/rate remains constant between energy harvests, i.e., the power/rate only potentially changes at an energy harvesting epoch.*

This lemma can be proved based on the concavity of function $f(p)$ in p. Due to space limitations here, the proof of this, and all upcoming lemmas will be omitted.

Therefore, in the following, we only consider policies where the rates are constant between any two consecutive energy arrivals. In order to simplify the notation, in this section, for any given T, we assume that there are $N-1$ energy arrival epochs (excluding $t = 0$) over $(0, T)$. We denote the last energy arrival epoch before T as s_{N-1}, and $s_N = T$, with $l_n = T - s_{n-1}$. Let us define (p_{1n}, p_{2n}) to be the transmit power over $[s_{n-1}, s_n)$.

Lemma 2 *For any feasible transmit power sequences $\mathbf{p}_1$, $\mathbf{p}_2$ over over $[0, T)$, the total number of bits departed from both users, denoted as B_1 and B_2, is a pentagon defined as*

$$\left\{ (B_1, B_2) \left| \begin{array}{l} B_1 \leq \sum_{n=1}^N f(p_{1n}) l_n \\ B_2 \leq \sum_{n=1}^N f(p_{2n}) l_n \\ B_1 + B_2 \leq \sum_{n=1}^N f(p_{1n} + g_{2n}) l_n \end{array} \right. \right\} \tag{7}$$

Lemma 3 *$\mathcal{D}(T)$ is a convex region. For any $T' > T$, $\mathcal{D}(T)$ is strictly inside $\mathcal{D}(T')$.*

As a first step, we aim to explicitly characterize $\mathcal{D}(T)$ for any T. Similar to the capacity region of the fading Gaussian multiple access channel [11], where each boundary point is a solution to $\max_{\mathbf{R}\in\mathcal{C}} \boldsymbol{\mu}\cdot\mathbf{R}$, here, in our problem, the boundary points also maximize $\boldsymbol{\mu}\cdot\mathbf{B}$ for some $\boldsymbol{\mu}$. First, let us examine three different cases separately.

A. $\mu_1 = \mu_2$.

In this subsection, we consider the scenario where $\mu_1 = \mu_2$. Therefore, our problem becomes $\max_{\mathbf{p}_1,\mathbf{p}_2} B_1 + B_2$. In [6], [7], we examined the optimal packet scheduling policy for the single-user scenario. We observe that for any fixed T, the optimal power allocation policy has the "majorization" property. Specifically, we have

$$i_n = \arg \min_{i_{n-1}<i\leq N} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \qquad (8)$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} \qquad (9)$$

In this two-user multiple access channel, if we want to maximize the sum of departures, we conclude that the sum of powers must have the same "majorization" property as in the single-user scenario. Therefore, we merge the energy arrivals from both users, and obtain the sum of energy arrivals as a function of t. We can obtain the optimal sequence of sum of transmit powers, p_1, p_2, …, p_n based on (8)-(9).

With the sum of powers fixed, we want to find feasible power allocations which maximize B_1 and B_2, individually. As we proved for the single-user case, whenever the sum of powers changes, the total amount of energy consumed up to that instance must be equal to the total amount of energy harvested up to that instance. In other words, both users must deplete their energy completely at that moment. This adds additional energy constraints on both users besides the causality constraints.

In order to maximize B_1, we plot the sum of E_{1n} as a function of t in Fig. 3. Then, we equalize the transmit powers of the first user as much as possible with the casuality constraints on energy and the additional energy consumption constraints. This latter constraint requires us to empty the energy queue at given instances s_{i_1}, s_{i_2}, etc. The former constraint requires us to choose the minimum slope among the lines passing through the origin and any other corner point before the next energy emptying epoch, [6], [7]. This gives us the sequence of p_{1n}, as shown in Fig. 3. Based on the concavity of the function $f(p)$, we can prove that this policy maximizes B_1 under the constraint that $B_1 + B_2$ is maximized at the same time.

Once p_{1n} is obtained, p_{2n} can be obtained by subtracting p_{1n} from p_n. This power allocation defines a pentagon region for (B_1, B_2), where the lower corner point of this pentagon is also the lower point on the flat part of the dominant face of $\mathcal{D}(T)$, which is point 1 in Fig. 4. Similarly, we can obtain

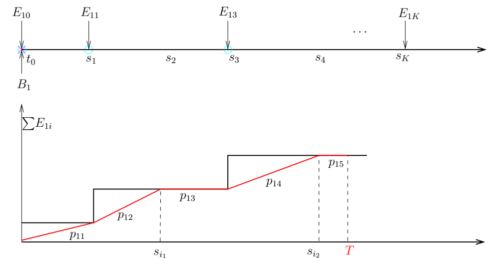

Fig. 3. The transmit powers of individual user.

the upper corner point on the flat part of the dominant face of $\mathcal{D}(T)$, which is point 2 in in Fig. 4. Since any linear combinations of these two policies still achieves the sum rate, any point on the flat part of the dominant face can be achieved. Therefore, the flat part of the dominant face of $\mathcal{D}(T)$ is bounded by these two corner points.

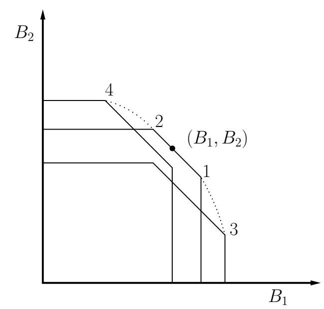

Fig. 4. The departure region $\mathcal{D}(T)$.

B. $\mu_1 = 0$ or $\mu_2 = 0$.

In this subsection, we aim to maximize the departure from one user only. This procedure is exactly the same as the procedure in the single-user scenario. On top of that, we also want to maximize the departure from the other user. Without loss of generality, we aim to maximize B_1 first. This is a single-user scenario, and the optimal policy can be obtained according to (8)-(9). Given the allocation p_{1n}^*, in order to maximize the departure from the second user, we need to solve the following optimization problem

$$\begin{aligned} \max_{\mathbf{p}_2} \quad & \sum_{n=1}^{N} f(p_{1n}^* + p_{2n}) l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad 1 \leq j \leq N \end{aligned} \qquad (10)$$

Theorem 1 *The optimal power allocation for (10) can be interpreted as a backward waterfilling process with base water level p_{1n}^* over $[s_{n-1}, s_n)$ for $1 \leq n \leq N$. Starting from $n = N$, we fill the energy $E_{2,N-1}$ over $[s_{N-1}, s_N)$, and get an updated water level as $p_{2N} + p_{1N}^*$; and then, we start to fill energy E_{N-2} over $[s_{N-2}, s_{N-1})$; once the water level exceeds $p_{2N} + p_{1N}^*$, we fill the remaining energy over $[s_{N-2}, s_N)$ until it is depleted. We continue this process until $n = 0$. The difference between the updated water level and base water level gives $\mathbf{p}_2$.*

The *backward waterfilling* procedure is shown in Fig. 5. This power allocation defines another pentagon, and its lower corner point maximizes B_1, which is point 3 in Fig. 4.

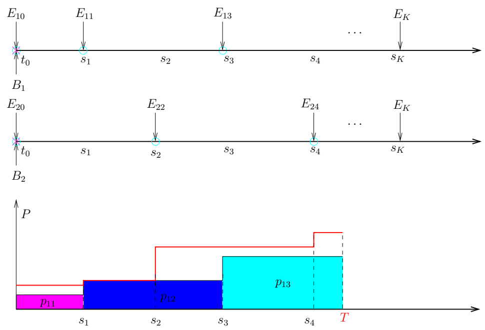

Fig. 5. The optimal transmit power for the second user to maximize its departure.

Similarly, we can obtain another pentagon whose upper corner point maximizes B_2, which is point 4 in Fig. 4. In general, points 3 and 4 do not coincide with the points 1 and 2, respectively, and consequently, there are curved parts connecting these corner points.

C. General $\mu_1, \mu_2 > 0$.

The curved parts can be characterized through the solution of $\max_{\mathbf{B}\in\mathcal{D}(T)} \boldsymbol{\mu}\cdot\mathbf{B}$ for some $\boldsymbol{\mu} > \mathbf{0}$. Since each boundary point corresponds to a corner point on some pentagon, for $\mu_1 > \mu_2$, we need to solve the following problem:

$$\begin{aligned} \max_{\mathbf{p}_1,\mathbf{p}_2} \quad & (\mu_1-\mu_2)\sum_n f(p_{1n})l_n + \mu_2 \sum_n f(p_{1n}+p_{2n})l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n}l_n \le \sum_{n=0}^{j-1} E_{1n}, \quad \forall j: 0<j\le N \\ & \sum_{n=1}^{j} p_{2n}l_n \le \sum_{n=0}^{j-1} E_{2n}, \quad \forall j: 0<j\le N \end{aligned} \tag{11}$$

The problem in (11) is a convex optimization problem with linear constraints, therefore, the unique global solution satisfies the extended KKT conditions as follows:

$$\frac{\mu_1-\mu_2}{1+p_{1n}} + \frac{\mu_2}{1+p_{1n}+p_{2n}} \le \sum_{j=n}^{N}\lambda_j, \quad 1\le n\le N \tag{12}$$

$$\frac{\mu_2}{1+p_{1n}+p_{2n}} \le \sum_{j=n}^{N}\beta_j, \quad 1\le n\le N \tag{13}$$

where the conditions in (12) and (13) are satisfied with equality if $p_{1n}, p_{1n} > 0$. When $\mu_1 \ne \mu_2$, it is difficult to obtain the optimal policy explicitly from the KKT conditions. Therefore, we adopt the idea of *generalized iterative waterfilling* in [12] to find the optimal policy.

Specifically, given the power allocation of the second user, denoted as $\mathbf{p}_2^*$, we optimize the power allocation of the first user, i.e., we aim to solve the following problem:

$$\begin{aligned} \max_{\mathbf{p}_1} \quad & (\mu_1-\mu_2)\sum_{n=1}^{N} f(p_{1n})l_n + \mu_2 \sum_{n=1}^{N} f(p_{1n}+p_{2n}^*)l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n}l_n \le \sum_{n=0}^{j-1} E_{1n}, \quad 0<j\le N \end{aligned} \tag{14}$$

Once the power allocation of the first user is obtained, denoted as $\mathbf{p}_1^*$, we do a *backward waterfilling* for the second user to obtain its optimal power allocation. We perform the optimization for both users in an alternating way. Because of the concavity of the objective function and the Cartesian product form of the convex constraint set, it can be shown that the iterative algorithm converges to the global optimal solution [13].

Because there is more than one term in the objective function of (14), the optimal policy for the first user does not have a backward waterfilling interpretation. However, using the method in [12], we can interpret the procedure for the first user as a *generalized backward waterfilling* operation. In order to see that, given $\mathbf{p}_2^*$, we define a generalized water level $b_n(p_{1n})$ as the inverse of the left hand side of (12), i.e.,

$$b_n(p_{1n}) = \left(\frac{\mu_1-\mu_2}{1+p_{1n}} + \frac{\mu_2}{1+p_{1n}+p_{2n}^*}\right)^{-1} \tag{15}$$

and the base water level as $b_n(0)$, which can be seen as the modified interference plus noise level over the duration $[s_{n-1}, s_n)$. We generalize the form of the water level by taking the priority of users into account. Then, the KKT condition for this single-user problem is

$$\frac{1}{b_n(p_{1n})} \le \sum_{j=n}^{N}\tilde{\lambda}_j, \quad n=1,2,\ldots,N \tag{16}$$

We note that $\tilde{\lambda}_j$ in general is different from the Lagrange multiplier λ_j in (12), since p_{2n}^* need not be the optimal $\mathbf{p}_2$. However, because of the convergence of the iterative algorithm, $\tilde{\lambda}_j$ converges to λ_j eventually as well.

Therefore, under the definition of the generalized water level $b_n(p_{1n})$, we can also interpret the optimal solution for the first user as a *generalized backward waterfilling* process. We first fill $E_{1,N-1}$ over the duration $[s_{N-1}, s_N)$, with the base water level $b_N(0)$. This step gives us an updated water level $b_N(E_{1,N-1}/l_N)$. Then, we move backward to the duration $[s_{N-2}, s_{N-1})$, and fill $E_{1,N-2}$ over that duration until it is depleted, or the water level becomes equal to $b_N(E_{1,N-1}/l_N)$. Once the latter happens, we fill the remaining energy over the durations $[s_{N-2}, s_{N-1})$ and $[s_{N-1}, s_N)$ in a way that the water level always becomes even. We repeat the steps until E_{10} is finished. This allocation gives the optimal $\mathbf{p}_1$ when the power of the second user is fixed.

In this section, we determined the largest (B_1, B_2) region for any given T, i.e., $\mathcal{D}(T)$. In the next section, we go back to our original problem, which is to minimize T for a given (B_1, B_2), and solve it, using our findings in this section.

IV. Minimizing the Transmission Duration: Minimizing T for a Given (B_1, B_2)

For a given pair (B_1, B_2), in order to minimize the transmission completion time of both users, we need to obtain T such that (B_1, B_2) lies on the boundary of the departure region $\mathcal{D}(T)$, as shown in Fig. 4. However, $\mathcal{D}(T)$ depends on T, which is the objective we want to minimize, and is unknown upfront.

Therefore, in order to solve the problem, we first calculate $\mathcal{D}(t)$ for $t = s_1, s_2, \ldots, s_K$. Then, we locate (B_1, B_2) on the maximum departure region. If (B_1, B_2) is exactly on the boundary of $\mathcal{D}(t)$ for some $t = s_i$, then, based on the "duality" of these two problems, we know that this s_i is exactly the minimum transmission completion time the system can achieve, and the corresponding power and rate allocation policy achieving this point is the optimal policy.

If (B_1, B_2) is outside $\mathcal{D}(s_i)$ but inside $\mathcal{D}(s_{i+1})$ for some s_i, then, we conclude that the minimum transmission completion time, T, must lie between these two energy arriving epoches, i.e., $s_i < T < s_{i+1}$. Therefore, $T - s_i$, denoted as t here, is the duration we aim to minimize.

We propose to solve this optimization problem in two steps. In the first step, we aim to find a set of power allocation policy to ensure that (B_1, B_2) is on the boundary of the departure region defined by this power allocation policy. In the second step, with the power allocation obtained in the first step, we find a set of rate allocation within its corresponding capacity region, such that B_1, B_2 are finished by the minimal transmission duration obtained in the first step. The first step guarantees that such a rate allocation exists. Solving the problem through these two steps significantly reduces the complexity for each problem, since the number of unknown variables is about half in each problem. In addition, as we will observe, the first step can be formulated as a standard convex optimization problem, and the second step becomes a linear programming problem. Therefore, both steps can be solved through standard optimization tools in an efficient way.

Let us define the energy spent over $[s_{n-1}, s_n)$ by the first and second transmitter as e_{1n}, e_{2n}, respectively. Then, let $\mathbf{e}_1 = [e_{11}, e_{1n}, \ldots, e_{1,i+1}]$, and $\mathbf{e}_2 = [e_{21}, e_{2n}, \ldots, e_{2,i+1}]$, we formulate the optimization problem in the first step as follows

$$
\begin{aligned}
\min_{\mathbf{e}_1, \mathbf{e}_2, t} \quad & t \\
\text{s.t.} \quad & \sum_{n=1}^{j} e_{1n} \leq \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \leq i+1 \\
& \sum_{n=1}^{j} e_{2n} \leq \sum_{n=0}^{j-1} E_{2n}, \quad 0 < j \leq i+1 \\
& B_1 \leq \sum_{n=1}^{i} f\left(\frac{e_{1n}}{l_n}\right) l_n + f\left(\frac{e_{1,i+1}}{t}\right) t \\
& B_2 \leq \sum_{n=1}^{i} f\left(\frac{e_{2n}}{l_n}\right) l_n + f\left(\frac{e_{2,i+1}}{t}\right) t \\
& B_1 + B_2 \leq \sum_{n=1}^{i} f\left(\frac{e_{1n} + e_{2n}}{l_n}\right) l_n \\
& \qquad + f\left(\frac{e_{1,i+1} + e_{2,i+1}}{t}\right) t \qquad (17)
\end{aligned}
$$

where the last three inequality constraints simply mean that $(B_1, B_2) \in \mathcal{D}(s_i + t)$. We state the problem in this form, so that the constraint set becomes convex, and the problem is transformed into a standard convex optimization problem. The joint concavity of $f\left(\frac{e}{t}\right) t$ in (e, t) can be proved through taking second derivatives of the function with respect to e and t, and observing that the Hessian is always negative semidefinite. Therefore, the right hand side of these inequality constraints are all jointly concave, thus the constraint set is convex.

Once we obtain $\mathbf{e}_1, \mathbf{e}_2$ and t, we divide the energy by its corresponding duration, and get the optimal power policy sequences $\mathbf{p}_1$ and $\mathbf{p}_2$. Next, we perform the rate allocation in the second step. Therefore, the problem becomes that of searching for $\mathbf{r}_1$ and $\mathbf{r}_2$ from the sequence of capacity regions defined by the sequences $\mathbf{p}_1$ and $\mathbf{p}_2$ to depart B_1 and B_2. This solution may not be unique. Therefore, we formulate it as a linear programming problem as follows:

$$
\begin{aligned}
\min_{\mathbf{r}_1, \mathbf{r}_2} \quad & r_{1,i+1} \\
\text{s.t.} \quad & \sum_{n=1}^{i} r_{1n} l_n + r_{1,i+1} t = B_1 \\
& \sum_{n=1}^{i} r_{2n} l_n + r_{2,i+1} t = B_2 \\
& (r_{1n}, r_{2n}) \in \mathcal{C}(p_{1n}, p_{2n}), \quad 0 < n \leq i+1 \quad (18)
\end{aligned}
$$

Here the objective function can be any arbitrary linear function in $\mathbf{r}_1$ and $\mathbf{r}_2$, since our purpose is only to obtain a feasible solution satisfying the constraints. We choose the objective function to be $r_{1,i+1}$ for simplicity. The solution of the optimization problem (17)-(18) gives us an optimal power and rate allocation policies, which minimize the transmission completion time for both users.

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