Optimal Sensing Scheduling in Energy Harvesting Sensor Networks

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***Abstract*—In this paper, we consider a collaborative sensing scenario where sensing nodes are powered by energy harvested from the environment. In each time slot, an active sensor consumes one unit amount of energy to take an observation and transmit it back to a fusion center (FC). After receiving observations from all of the active sensors in a time slot, the FC aims to extract information from them. We assume that the utility generated by the observations is a function of the number of the active sensing nodes in that slot. Assuming the energy harvesting processes at individual sensors are independent Bernoulli processes, our objective is to develop a sensing scheduling policy so that the expected long-term average utility generated by the sensors is maximized. Under the concavity assumption of the utility function, we first show that the expected time average utility has an upper bound for any feasible scheduling policy satisfying the energy causality constraint. We then propose a myopic policy, which aims to select a fixed number of sensors with the highest energy levels to perform the sensing task in each slot. The myopic policy essentially balances the current energy queue lengths in every time slot. We show that the time average utility generated under the myopic policy converges to the upper bound almost surely as time T approaches infinity, thus the myopic policy is optimal. The corresponding convergence rate is also explicitly characterized.**

I. Introduction

Sensor networks equipped with energy harvesting devices have attracted great attentions recently. Compared with conventional sensor networks powered by batteries, the energy harvesting abilities of the sensor nodes make sustainable and environment-friendly sensor networks possible. Such renewable energy supply feature also necessitates a completely different approach to energy management.

Under an energy harvesting setting, energy management schemes have been developed to optimize communication related metrics, such as channel capacity, transmission delay or network throughput [1]–[8], and signal processing related performance metrics, such as estimation mean square error, detection delay, false alarm probability [9], [10].

In this paper, we focus on the design of a collaborative sensing scheduling scheme in a sensor network powered by energy harvested from the environment. Our motivation is a collaborative sensing scenario where multiple sensors are deployed to monitor the status of a phenomenon in a region. While collaborative sensing schemes have been well studied under a conventional battery-powered setting, to the best of our knowledge, the optimal sensing scheduling for rechargeable sensing nodes has not been studied before. Our objective is to coordinate the sensing actions among multiple sensor nodes in a way that the time average sensing performance (such as the probability of detection error) is optimized. Our primary constraint is the energy causality constraint at each sensor.

Specifically, we assume that a sensor takes a unit of energy to sense the nature and send its observation to a fusion center (FC). Sensors cannot perform the sensing task when there is not sufficient energy in its battery. The FC combines the observations collected from sensors and extracts information from them. We assume that in each slot, the *utility* generated by those observations is a function of the set of active sensors in that slot. Our objective is to select a subset of sensors to perform the sensing task in each time slot, such that the long-term average utility is optimized, while the energy constraint at each individual sensor is satisfied at every time slot. The problem has a combinatorial nature and is hard to solve in general. The randomness of the energy harvesting processes at sensors makes the problem even more challenging.

To make the problem tractable, we assume that the utility function is symmetric with respect to sensors, i.e., it is a function of the total number of active sensors in each slot. In addition, we assume that it is a concave function defined over $\mathbb{Z}_+$. Under these assumptions, we show that the expected time average utility has an upper bound for any feasible scheduling policy satisfying the energy causality constraint. We then propose a myopic policy, which aims to select a fixed number of active sensors with the *longest energy queues* in each slot. Under Bernoulli assumptions of the energy harvesting processes at individual sensors, we show the time average utility generated under the myopic policy converges to the upper bound almost surely as time T approaches infinity, thus the myopic policy is optimal. The corresponding convergence rate is characterized explicitly.

We point out that similar queue-length based scheduling policies have been well investigated in queueing theory, e.g., [11], [12]. Reference [11] proposes an algorithm to maximize the throughput of a single-hop system with N parallel queues and one server. The algorithm is to allocate the server to the longest connected queue at any given time slot. [12] generalizes [11] by considering a system with N queues and K servers. The optimal policy to minimize delay is to assign the servers to the K longest connected queues.

Although the myopic policy proposed in this paper has a similar queue-length balancing nature, the problem studied

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here is fundamentally different from [11], [12]. Essentially, the queue-length balancing structure of the optimal policies in [11], [12] is due to the constraints on the server side, i.e., only a fixed number of queues can be served in each time slot, one for each server. In order to maximize the efficiency of the system, queue lengths should be well balanced so that the probability that every server has a non-empty queue to serve is maximized. In our paper, we do not have such constraints on "servers", i.e., all sensors can be active simultaneously. Rather, the structure of our policy is due to two factors: the concavity of the utility function, and the energy causality constraints on sensors. The concave property of the utility function requires us to equalize the number of active sensors in each slot, while the energy causality constraints on sensors determine that in order to maximally equalize the number of active sensors in each slot, we should select the nodes with the longest energy queues for sensing in every slot. The techniques we use to prove the optimality of the myopic policy is also different from those in [11], [12].

II. System Model and Problem Formulation

In this paper, we consider a sensor network consisting of N sensors (randomly) distributed in an area. Each sensor node is powered by energy harvested from ambient environment. We assume that each sensor node has an energy queue to store the harvested energy. The energy queue has a maximum storage capacity E_{max}. For now, we consider the case where $E_{max} = +\infty$. The energy queue at each sensor is replenished randomly and consumed by taking observations and transmitting them to a fusion center (FC). We assume that a unit amount of energy is required for one sense-and-transmit operation.

We consider a time-slotted system. In time slot t, a subset of sensors, denoted as $\mathcal{C}_t$, is selected to sense the environment, and transmit their observations to the FC. We assume a sensor can make at most one observation in each slot. The FC then combines the observations for inference. The utility generated by those observations is a function of $\mathcal{C}_t$, denoted as $f(\mathcal{C}_t)$. The total sensing utility over duration $[1,T]$ is simply the sum of the utilities generated in each slot in $[1,T]$. We make the following assumptions on the utility function $f(\mathcal{C}_t)$.

Assumptions 1

(0) *$f(\mathcal{C})$ is a function of the size of $\mathcal{C}$, i.e., $f(\mathcal{C}) = f(|\mathcal{C}|)$.*
(i) *$f(m)$ is monotonically increasing in m.*
(ii) *$f(m+1) + f(m-1) < 2f(m)$ for $m \in \mathbb{Z}_+$.*

Assumption 1-(0) implies that $f(\mathcal{C})$ is symmetric with respect to sensor nodes. By imposing this assumption, we essentially ignore the differences in contributions from different sensing nodes, and focus on the impact of the total number of collected observations on the sensing performance. Assumption 1-(i) means that the utility function increases as more observations are collected. Assumption 1-(ii) essentially means that $f(m)$ is a concave function defined over $\mathbb{Z}_+$. These assumptions are quite general and reasonable, and they are satisfied by many utility functions.

Let $E_i(t)$ denote the amount of energy remaining in the battery of node i at the beginning of time slot t, $A_i(t)$ be the amount of harvested energy at node i during slot t. Then, the energy queue evolves according to

$$E_i(0) = 0, \forall i$$
$$E_i(t+1) = E_i(t) - \mathbf{1}_{i\in\mathcal{C}_t} + A_i(t), \quad t = 0,1,2,\ldots, \forall i \quad (1)$$

where $\mathbf{1}_x$ is an indicator function, i.e., it equals one if x is true, and it equals zero otherwise. Since an observation cannot be made if $E_i(t) < 1$, we impose the following energy constraints

$$E_i(t) \geq \mathbf{1}_{i\in\mathcal{C}_t}, \qquad \forall i, t. \quad (2)$$

We assume that energy arrives randomly in each time slot. Specifically, for every sensor node i, we assume the energy arrival process is a Bernoulli process with parameter λ_i, $0 \leq \lambda_i \leq 1$, i.e., $\mathbb{E}[A_i(t)] = \lambda_i$. The arrival processes are independent across sensors. We assume $\sum_{i=1}^N \lambda_i$ is an *integer* for a clear exposition of the analysis. For a general setting where $\sum_{i=1}^N \lambda_i$ may be a non-integer, the corresponding scheduling policy and analysis will be slightly different, and can be found in [13].

Our objective is to select the set of sensors $\mathcal{C}_t$ to perform the sensing task in each time slot, such that the expected long-term average utility is maximized, subject to the energy constraint for each individual sensor at every time slot. The optimization problem is formulated as

$$\max_{\{\mathcal{C}_t\}} \quad \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \quad (3)$$
$$\text{s.t.} \quad (1)-(2)$$

III. The Optimal Sensing Scheduling

The optimization problem in (3) is stochastic and has a combinatorial nature, thus it is in general hard to solve. However, with Assumption 1, we first show that the optimal solution has an upper bound, which corresponds to a scheduling policy with a fixed number of active sensors in every slot. Motivated by this observation, we then propose a myopic policy, which greedily selects a subset of sensors with the longest energy queues to perform the sensing task in each slot. We prove its optimality by showing that the myopic policy asymptotically achieves the upper bound.

A. An upper bound

Definition 1 *A sensing scheduling policy $\{\mathcal{C}_t\}_t$ is feasible if $E_i(t) \geq 1$, for every $i \in \mathcal{C}_t$, $\forall t$, i.e., the energy causality constraint (2) is always satisfied for every i,t.*

Lemma 1 *Under every feasible scheduling policy, we have*

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{i\in\mathcal{C}_t} \leq \lambda_i, \quad a.s. \quad \forall i \quad (4)$$

Proof: Lemma 1 can be proved based on the energy queue evolution described in (1) and the definition of feasible scheduling policy. Since $E_i(t) - \mathbf{1}_{i\in\mathcal{C}_t} \geq 0$ for every $t \geq 1$, we have

$$\sum_{t=1}^{T} \mathbf{1}_{i\in\mathcal{C}_t} \leq \sum_{t=0}^{T-1} A_i(t).$$

Therefore,

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{i\in\mathcal{C}(t)} \leq \limsup_{T\to+\infty} \frac{1}{T}\sum_{t=0}^{T-1} A_i(t) = \lambda_i, \quad a.s.$$

where the last equality follows from the strong law of large numbers. ■

Lemma 1 implies that for any feasible scheduling policy $\{\mathcal{C}_t\}$, the long-term fraction of time slots that a sensor is active must be upper bounded by the energy arrival rate at that sensor. This is an intuitive result due to the energy causality constraint. Lemma 1 motivates us to obtain an upper bound on the objective function in (3) by removing the energy causality constraint in (2), and impose a relaxed energy constraint, i.e., the average energy constraint in (4) instead.

Lemma 2 *The objective function in (3) is upper bounded as*

$$\max_{\{\mathcal{C}_t\}} \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \leq f\left(\sum_{i=1}^{N}\lambda_i\right). \tag{5}$$

Proof: We prove Lemma 2 based on the properties of $f(\mathcal{C})$ assumed in Assumption 1. Specifically, we have

$$\begin{aligned}
&\max_{\{\mathcal{C}_t\}} \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \\
&\leq \max_{\{\mathcal{C}_t\}} \limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \\
&\leq \max_{\{\mathcal{C}_t\}} \mathbb{E}\left[\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] && (6)\\
&\leq \max_{\{\mathcal{C}_t\}} \mathbb{E}\left[f\left(\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} \mathcal{C}_t\right)\right] && (7)\\
&\leq f\left(\sum_{i=1}^{N}\lambda_i\right) && (8)
\end{aligned}$$

where (6) follows from Fatou's Lemma, and (7) follows from the concavity and monotonicity of function f. Applying Lemma 1, we have (8) hold. ■

The upper bound in (5) desires a uniform sensing scheduling, i.e., in order to maximize the long-term average utility, in each time slot, the scheduler should select $\sum_{i=1}^{N}\lambda_i$ sensor nodes to perform the sensing task. The selection should be coordinated in a way to ensure that, with high probability, there exists at least $\sum_{i=1}^{N}\lambda_i$ nodes with non-empty energy queues (i.e., $E_i(t) \geq 1$) in every time slot. The randomness of the energy arrival processes makes such coordination non-trivial. For a network with identical energy harvesting statistics for all sensors (i.e., λ_is are equal), the optimal scheduling is quite intuitive: Sensor nodes with higher energy levels should be utilized in the current slot, since their probabilities to become empty in future slots are relative low. Thus, $\sum_{i=1}^{N}\lambda_i$ sensor nodes with the longest energy queues should be selected in each slot. However, when λ_is are not equal, the optimal scheduling is not quite straightforward. For this case, the energy queue length may not accurately indicate the probability of a sensor becoming empty in the future. There are possibilities that sensors have larger λ_i may have shorter queue lengths in certain time slots, due to fluctuations in the energy harvesting processes. Thus, the sensor selection should jointly consider the current energy queue length information as well as the energy arrival rate for each sensor, which makes the problem very complicated. However, as we will show in the following section, greedily selecting $\sum_{i=1}^{N}\lambda_i$ sensor nodes with the longest energy queues for sensing is still optimal, even if λ_is are not equal.

B. A myopic policy

Motivated by the upper bound in Lemma 2, and the intuition to balance the energy queue lengths for the purpose of reducing the probability that energy queues become empty, we propose a myopic policy as follows.

At the beginning of time slot t, the system first selects $\sum_{i=1}^{N}\lambda_i$ nodes with the longest energy queues and form a candidate set of active sensors, denoted as $\mathcal{C}'_t$. Then, the scheduling policy $\{\mathcal{C}^*_t\}$ is determined as

$$\mathcal{C}^*_t = \{i : i \in \mathcal{C}'_t, E_i(t) \geq 1\}.$$

Selecting $\{\mathcal{C}^*_t\}$ in this way guarantees that the myopic policy is always feasible.

Theorem 1 *The myopic policy $\{\mathcal{C}^*_t\}$ is optimal, i.e.,*

$$\liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}^*_t)\right] = f\left(\sum_{i=1}^{N}\lambda_i\right).$$

Theorem 2 *Under the myopic scheduling policy, for any sufficiently large T, we have*

$$\mathbb{P}\left[\frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{|\mathcal{C}^*_t| \neq \sum_{i=1}^{N}\lambda_i} \geq \epsilon\right] \leq 2T^2 \exp\left(-\frac{T\epsilon^2}{12Nm^2}\right)$$

$$\mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}^*_t) - f\left(\sum_{i=1}^{N}\lambda_i\right)\right| \geq \epsilon\right] \leq 2T^2 \exp\left(-\frac{T\epsilon^2}{12NM^2m^2}\right)$$

where $m := \sum_{i=1}^{N}\lambda_i$, $M := f\left(\sum_{i=1}^{N}\lambda_i\right) - f(0)$.

The proof of Theorem 1 is provided in Appendix. The proof of Theorem 2 is omitted for the brevity of this paper.

Theorem 1 indicates that the expected average utility generated under the myopic policy converges to the upper bound, thus it is optimal. Theorem 2 implies that in almost every time slot, we have $|\mathcal{C}^*_t| = \sum_{i=1}^{N}\lambda_i$, and the time average utility generated under the myopic policy converges to the upper bound almost surely. The corresponding convergence rates are explicitly characterized.

IV. Simulation Results

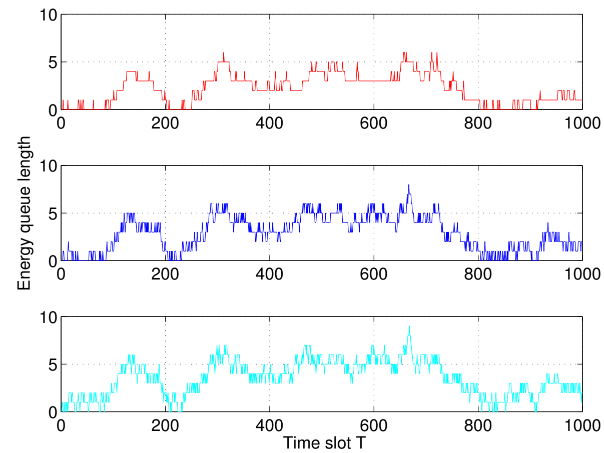
In this section, we evaluate the performance of the proposed myopic scheduling algorithm through simulations.

To illustrate the temporal evolution of the energy queue lengths and the scheduling procedure under the myopic algorithm, we first consider a small sensor network consisting of 3 sensor nodes. The energy arrival rates for sensors are $\lambda_1 = 0.1$, $\lambda_2 = 0.3$, $\lambda_3 = 0.6$. The myopic algorithm is thus to select one sensor with the longest energy queue length to perform the sensing task in each time slot. Starting with an empty initial state, one sample path of the energy queue evolution for each sensor is plotted in Fig. 1(a). For a time slot with $|\mathcal{C}_t^*| \neq \sum_{i=1}^N \lambda_i$, we call it an *unsaturated* time slot; otherwise, we call it a *saturated* time slot. The fraction of saturated time slots (i.e., $|\mathcal{C}_t^*| = 1$) up to T is plotted as a function of T in Fig. 1(b). We observe that the energy queue lengths of those three sensor nodes are closely coupled together. The differences in queue lengths are small for most of the time slots, and the queue lengths fluctuate in the same manner in time. This coincides with our objective to balance the queue lengths through the myopic scheduling policy. In Fig. 1(b), we observe that the sample path-wise fraction of saturated time slot approaches 1 as T increases. Although this fraction fluctuates significantly when T is small, it becomes smooth as T increases, and gradually approaches 1. This indicates that under the myopic scheduling policy, for a sufficiently large T, the system has one active node to perform the sensing task for almost every time slot in $[1, T]$, which is the best we can hope for. The result in Fig. 1(b) implies the effectiveness of balancing energy queues in maximizing the time-average utility function.

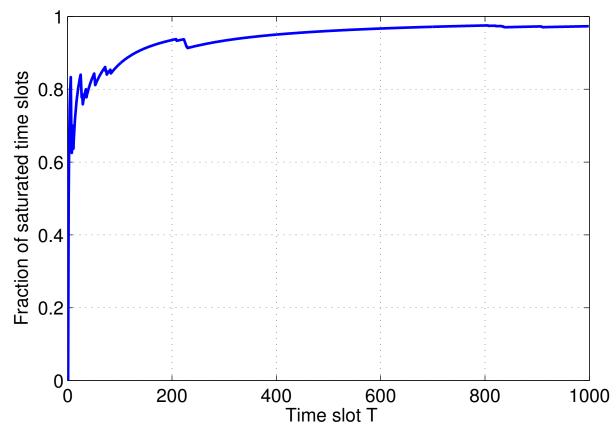
We then consider the empirical average performances of the proposed scheduling policy. We first consider a sensor network with $N = 60$. We consider three different energy harvesting profiles for the network. We let the energy harvesting rate vectors be $\boldsymbol{\lambda}_1 = [0.1 \times \text{ones}(1,30), 0.9 \times \text{ones}(1,30)]$, $\boldsymbol{\lambda}_2 = [0.1 \times \text{ones}(1,15), 0.9 \times \text{ones}(1,15), 0.5 \times \text{ones}(1,30)]$, $\boldsymbol{\lambda}_3 = [0.5 \times \text{ones}(1,60)]$, respectively. We select the rate vectors in this way to make sure that the desired number of active sensors in each slot is 30 under each setup. However, since the rate vectors are different, the statistics of the energy harvesting processes at individual sensors differ under those setups. We randomly pick 1000 energy harvesting sample paths for each $\boldsymbol{\lambda}_i$, and plot the average fraction of saturated time slots (i.e., $|\mathcal{C}_t^*| = 30$) up to T under the myopic policy in Fig. 2. We observe that for all of the three setup, the average fraction of saturated time slots monotonically increases as T increases, and gradually approaches 1. All of the three curves are concave in T. This simulation results verifies Theorem 1, i.e., the empirical mean of the time average utility function approaches $f(30)$ as T goes to infinity.

We also observe that among those three setups, the curve corresponding to $\boldsymbol{\lambda}_1$ is always at the bottom, while the curve corresponding to $\boldsymbol{\lambda}_3$ is always on the top. This phenomenon can be explained in this way: among $\boldsymbol{\lambda}_i, i = 1, 2, 3$, $\boldsymbol{\lambda}_3$ is the most evenly distributed vector while $\boldsymbol{\lambda}_1$ is the least evenly distributed one. Since the energy harvesting rates of the nodes are the same with $\boldsymbol{\lambda}_3$, intuitively, the myopic policy minimizes the probability that $|\mathcal{C}_t^*| < 30$ for every t. However, when the energy harvesting processes at sensors vary significantly, as with $\boldsymbol{\lambda}_1$, even though the myopic policy equalizes the current energy queue lengths as much as possible, it cannot guarantee that the probability that $|\mathcal{C}_t^*| < 30$ is minimized. Thus, the energy queues have a greater chance to become empty in this case, which is indicated by the lower average fraction of saturated time slots. We note that the gaps between the curves diminish as t increases. This implies that although the myopic policy may not always minimize the probability that $|\mathcal{C}_t^*| < 30$, the fraction of unsaturated time slots still converges to zero as T increases, therefore the myopic policy is still optimal.

At last, we fix the energy arrival rate at each sensor node to be 0.5, and vary the size of the sensor networks to be $N = 20, 40, 80$. We again run 1000 samples paths for each



(a)



(b)

Fig. 1: A sensor network with $N = 3$, $\lambda_1 = 0.1$, $\lambda_2 = 0.3$, $\lambda_3 = 0.6$. Fig. 1(a) plots a sample path of the energy queue lengths. Fig. 1(b) plots the corresponding fraction of saturated time slots up to T as a function of T.

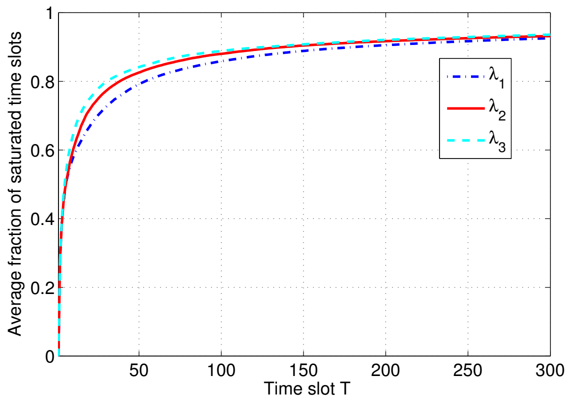


Fig. 2: The average fraction of saturated time slots as a function of time index T. $N = 60$, $\boldsymbol{\lambda}_1 = [0.1 \times \text{ones}(1, 30), 0.9 \times \text{ones}(1, 30)]$, $\boldsymbol{\lambda}_2 = [0.1 \times \text{ones}(1, 15), 0.9 \times \text{ones}(1, 15), 0.5 \times \text{ones}(1, 30)]$, $\boldsymbol{\lambda}_3 = [0.5 \times \text{ones}(1, 60)]$.

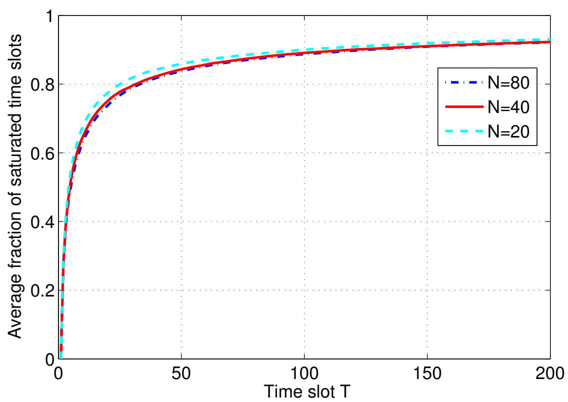


Fig. 3: The average fraction of saturated time slots as a function of time index T. $\lambda_i = 0.5, \forall i$, $N = 20, 40, 80$, respectively.

setup, and plot the average fraction of saturated time slots (i.e., $|\mathcal{C}_t^*| = 0.5N$) under the myopic policy in Fig. 3. We observe that among those three curves, the curve corresponding to $N = 80$ is always at the bottom, while the curve corresponding to $N = 20$ is always on the top. This is consistent with the theoretical results in Theorem 2, i.e., for a fixed T, the fraction of unsaturated time slots increases in N.

Appendix: Proof of Theorem 1

Before we proceed, we first introduce Hoeffding's inequality, which will be used repeatedly in the proof.

Theorem 3 (Hoeffding's inequality [14]) *Let $X_1, X_2, \ldots, X_n$ be independent bounded random variables such that $X_i \in [a_i, b_i]$ with probability 1. Let $S_n = \sum_{i=1}^n X_i$. Then for any $\epsilon > 0$, we have*

$$P\left(|S_n - \mathbb{E}(S_n)| \geq \epsilon\right) \leq 2\exp\left(-\frac{2\epsilon^2}{\sum_{i=1}^n (b_i - a_i)^2}\right).$$

By Fatou's lemma, in order to prove Theorem 1, it suffices to prove that

$$\mathbb{E}\left[\liminf_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t^*)\right] = f\left(\sum_{i=1}^{N} \lambda_i\right). \tag{9}$$

The definition of $\mathcal{C}_t^*$ implies that $\mathcal{C}_t^* \subseteq \mathcal{C}_t'$, $|\mathcal{C}_t^*| \leq |\mathcal{C}_t'| = \sum_{i=1}^N \lambda_i$. Due to Assumption 1, when $|\mathcal{C}_t^*| = |\mathcal{C}_t'|$, $f(\mathcal{C}_t^*) = f\left(\sum_{i=1}^N \lambda_i\right)$; when $|\mathcal{C}_t^*| < |\mathcal{C}_t'|$, $f(\mathcal{C}_t^*) < f\left(\sum_{i=1}^N \lambda_i\right)$. Thus, in order to prove (9), it suffices to prove that

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{|\mathcal{C}_t^*| < \sum_{i=1}^N \lambda_i} = 0, \quad a.s. \tag{10}$$

At each time slot t, we reorder $E_i(t)$, $i = 1, 2, \ldots, N$ according to their values, and denote $E_{(i)}(t)$ as the i-th largest one among them. Let $m := \sum_{i=1}^N \lambda_i$. For a given T, we define T_1 as the largest time index with $T_1 \leq T$ such that $E_{(m)}(T_1) = 0$. Thus for any $t \in (T_1, T]$, we have $E_{(m)}(t) = 1$, which implies $|\mathcal{C}_t^*| = m$. Assuming the system starts with empty energy queues, T_1 always exists.

When $E_{(1)}(T_1) > 0$, we can always define T_0 as the smallest time index such that $E_{(1)}(T_0 + 1) = E_{(1)}(T_1)$. Thus, $T_0 < T_1$. For any energy queue, the Bernoulli arrival assumption ensures that the queue length in a slot deviates at most by one from its previous slot. This observation together with the empty initial state assumption implies that $E_{(1)}(T_0) = E_{(1)}(T_0 + 1) - 1$. Then, at time T_0, we must have

$$E_{(1)}(T_0) = E_{(2)}(T_0) = \ldots = E_{(m+1)}(T_0) = E_{(1)}(T_1) - 1$$

This is due to the fact that in order to have a jump for the longest queue length at the beginning of time slot $T_0 + 1$, the associated sensor should have the same amount of energy $E_{(1)}(T_0)$ at the beginning of time slot T_0, and does not sense in slot T_0. At the same time, there must exist additional m sensors with the same energy level to sense in slot T_0. Thus,

$$\sum_{i=1}^{N} E_i(T_0) \geq (m+1)[E_{(1)}(T_1) - 1]. \tag{11}$$

On the other hand, based on the definition of T_1, we have

$$\sum_{i=1}^{N} E_i(T_1) \leq (m-1)E_{(1)}(T_1). \tag{12}$$

Combining (11) and (12), we have

$$\sum_{i=1}^{N} E_i(T_1) - \sum_{i=1}^{N} E_i(T_0) \leq -E_{(1)}(T_1) + m + 1 \tag{13}$$

Based on the definition of $E_i(t)$ in (1), we have

$$\sum_{i=1}^{N} E_i(T_1) - \sum_{i=1}^{N} E_i(T_0) \geq \sum_{t=T_0}^{T_1-1}\left(\sum_{i=1}^{N} A_i(t) - m\right) \tag{14}$$

$$\sum_{i=1}^{N} E_i(T) - \sum_{i=1}^{N} E_i(T_1) \leq \sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t) - m\right) + m \tag{15}$$

Then,

$$
\begin{aligned}
&\mathbb{P}\left[\sum_{i=1}^{N} E_i(T) > T\epsilon\right] \\
&\leq \mathbb{P}\left[\sum_{i=1}^{N} E_i(T_1)+\sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t)-m\right)+m > T\epsilon\right] \quad (16)\\
&\leq \mathbb{P}\left[mE_{(1)}(T_1)+\sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t)-m\right)+m > T\epsilon\right] \quad (17)\\
&\leq \mathbb{P}\Bigg[mE_{(1)}(T_1)+\sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t)-m\right) > T\epsilon - m,\\
&\qquad E_{(1)}(T_1) \leq \frac{T\epsilon}{2m}\Bigg] + \mathbb{P}\left[E_{(1)}(T_1) > \frac{T\epsilon}{2m}\right]\\
&\leq \mathbb{P}\left[\sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t)-m\right) > \frac{T\epsilon}{2} - m\right]\\
&\quad + \mathbb{P}\left[E_{(1)}(T_1) > \frac{T\epsilon}{2m}\right]
\end{aligned}
$$

where (16) follows from (15), (17) follows from (12). Note

$$
\begin{aligned}
&\mathbb{P}\left[\sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t)-m\right) > \frac{T\epsilon}{2} - m\right]\\
&= \sum_{t_1=1}^{T-1}\mathbb{P}\left[\sum_{t=T_1}^{T-1}\left(\sum_{i=1}^{N} A_i(t)-m\right) > \frac{T\epsilon-2m}{2}, T_1 = t_1\right]\\
&\leq \sum_{t_1=1}^{T-1}\mathbb{P}\left[\sum_{t=t_1}^{T-1}\sum_{i=1}^{N}(A_i(t)-\lambda_i) > \frac{T\epsilon-2m}{2}\right]\\
&\leq \sum_{t_1=1}^{T-1} 2\exp\left(-\frac{(T\epsilon-2m)^2}{2(T-t_1-1)N}\right) \quad (18)\\
&\leq 2(T-1)\exp\left(-\frac{(T\epsilon-2m)^2}{2TN}\right) \quad (19)
\end{aligned}
$$

where (18) follows from Hoeffding's inequality. Besides,

$$
\begin{aligned}
&\mathbb{P}\left[E_{(1)}(T_1) > \frac{T\epsilon}{2m}\right]\\
&= \mathbb{P}\left[-E_{(1)}(T_1)+m+1 < -\frac{T\epsilon}{2m}+m+1\right]\\
&\leq \mathbb{P}\left[\sum_{t=T_0}^{T_1-1}\left(\sum_{i=1}^{N} A_i(t)-m\right) < -\frac{T\epsilon}{2m}+m+1\right] \quad (20)\\
&\leq \sum_{t_0=1}^{T-1}\sum_{t_1=t_0+1}^{T-1}\mathbb{P}\left[\sum_{t=t_0}^{t_1-1}\sum_{i=1}^{N}(A_i(t)-\lambda_i) < -\frac{T\epsilon}{2m}+m+1\right]\\
&\leq \sum_{t_0=1}^{T-1}\sum_{t_1=t_0+1}^{T-1} 2\exp\left(-\frac{(T\epsilon-2m(m+1))^2}{2(t_1-t_0)Nm^2}\right) \quad (21)\\
&\leq (T-1)(T-2)\exp\left(-\frac{(T\epsilon-2m(m+1))^2}{2TNm^2}\right) \quad (22)
\end{aligned}
$$

where (20) follows from (13) and (14), (21) follows from Hoeffding's inequality. When T is sufficiently large, we have

$$(19) \leq 2(T-1)\exp\left(-\frac{T\epsilon^2}{3N}\right) \quad (23)$$

$$(22) \leq (T-1)(T-2)\exp\left(-\frac{T\epsilon^2}{3Nm^2}\right) \quad (24)$$

Combining (23) and (24), we have

$$\mathbb{P}\left[\sum_{i=1}^{N} E_i(T) > T\epsilon\right] \leq T^2\exp\left(-\frac{T\epsilon^2}{3Nm^2}\right) \quad (25)$$

Therefore, according to Borel-Cantelli lemma [15], we have

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{i=1}^{N} E_i(T) = 0, \quad a.s.$$

Based on (1) and the strong law of large numbers, we have

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}|\mathcal{C}_t^*| = \lim_{T\to+\infty}\frac{1}{T}\sum_{t=0}^{T}\sum_{i=1}^{N} A_i(t) = \sum_{i=1}^{N}\lambda_i, \quad a.s.$$

which implies (10) and completes the proof.

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