Adaptive Sensing Scheduling for Energy Harvesting Sensors with Finite Battery

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***Abstract*—In this paper, we study the optimal sensing scheduling policy for an energy harvesting sensing system equipped with a finite battery. The objective is to strategically select the sensing epochs such that the long-term average sensing performance is optimized. In the sensing system, it is assumed that the sensing performance depends on the time duration between two consecutive sensing epochs. Example applications include reconstructing a wide-sense stationary random process by using discrete-time samples collected by a sensor. The randomness of the energy harvesting process and the finite battery constraint at the sensor make the optimal sensing scheduling very challenging.**

Assuming the energy harvesting process is a Poisson random process, we first identify a performance limit on the long-term average sensing performance of the system without the finite battery constraint. We then propose an energy-aware adaptive sensing scheduling policy, which dynamically chooses the next sensing epoch based on the battery level at the current sensing epoch. We show that as the battery size increases, the sensing performance under the adaptive sensing policy asymptotically converges to the performance limit of the system with an infinite battery, thus it is asymptotically optimal. The convergence rate is also analytically characterized.

I. Introduction

In order to build a self-sustainable wireless sensor network, powering sensor nodes with energy harvesting devices becomes a natural and feasible solution. However, the random, scarce and non-uniform energy harvested from the ambient environment also necessitates a completely different approach to energy management. Different energy management schemes have been proposed to cope with the stochastic nature of harvested energy from different perspectives. Under the infinite battery assumption, optimal energy management policies have been proposed to maximize the communication and sensing performances under different settings [1]–[5]. However, when finite battery assumption is imposed, it changes the problem dramatically, and makes the corresponding optimal energy management much more complicated. One approach is to formulate the energy management problem as a one-shot offline optimization problem, under the assumption that the energy harvesting profile is known in advance. Examples include the throughput maximization problems studied in [6]–[8], where the the optimal policies are significantly different from their counterparts in an infinite battery setting [1], [2], [9]. Another approach is to formulate the optimal energy management problem as an online stochastic control problem, assuming that only the statistics and the history of the energy harvesting process are available at the controller. Modeling the energy replenishing process as a Markov process, [10] aims to maximize the time average reward by making decisions regarding whether to transmit or discard a packet based on the current energy level. The optimal policy is shown to have a threshold structure. [11] studies the performance limits of a sensing system where the battery size and the data buffer are finite and proposes an asymptotically optimal energy management scheme. The dynamic activation of sensors with unit battery in order to maximize the sensing utility is studied in [12]. In general, online optimal energy management policies under a finite battery constraint are often very difficult to characterize. Explicit solutions only exist for certain special scenarios.

This work was supported in part by the U.S. National Science Foundation (NSF) under Grants ECCS-1202075 and ECCS-1405403.

In this paper, we consider the optimal online sensing scheduling policy of an energy harvesting sensing system under finite battery constraint. We aim to investigate the impact of finite battery size on the sensing performance, and characterize the fundamental performance limits of the sensing system. Specifically, we assume that the energy harvesting process is a Poisson process, and each sensing operation consumes one unit amount of energy. Harvested energy enters the battery, and is then consumed by sensing operations. Battery overflow happens when the energy level exceeds the battery capacity. Assuming the sensing performance is a function of the durations between any two consecutive sensing epochs, our objective is to strategically place sensing epochs in a way so that the long-term average sensing performance is optimized. As a first step, we assume the sensing performance over a sensing duration $(0,T)$ is measured by $\sum_n f(d_n)$, where d_n is the duration between two consecutive sensing epochs. Moreover, we assume that $f(d)$ is convex and increasing in d, and $f(d)/d$ is increasing in d and upper bounded. One example application is to use samples collected at discrete time instances to estimate a time evolving physical quantity (temperature, humidity, etc), which can be modeled as a random process with power-law decaying covariance [4].

we first identify a lower bound on the long-term time average sensing performance under any sensing scheduling policy for the system without the finite battery constraint. We then propose an energy-aware adaptive sensing scheduling policy, which dynamically chooses the next sensing epoch based on the battery level at the current sensing epoch. We show that sensing scheduling policy achieves a performance close to that with an unlimited battery size.

The problem we consider is similar to that in [11], however, it is also significantly different from it. [11] considers a time-slotted system, while the objective is to vary the amount of energy spent in each time slot to optimize the system performance. However, we consider a continuous-time system in this paper, and the proposed asymptotically optimal design varies the sensing frequency in time. Since the durations between sensing epochs are not equal, it makes the analysis of the system performance under the proposed policy much more challenging.

II. System Model and Problem Formulation

A. Energy Harvesting Model

Consider a sensor node powered by energy harvested from the ambient environment. We assume that the sensor node has an energy queue, such as a rechargeable battery or a super capacitor, to store the harvested energy. The energy queue is replenished randomly and consumed by taking observations. It is assumed that a unit amount of energy is required for one sensing operation. Assume each sensor is equipped with a finite battery with capacity B.

The energy arrival follows a Poisson process with parameter 1. Hence, energy arrivals occur in discrete time instants. Specifically, we use $t_1, t_2, \ldots, t_n, \ldots$ to represent the energy arrival epochs. Then, the energy inter-arrival times $t_i - t_{i-1}$ are exponentially distributed with mean 1. Without loss of generality, it is assumed that the system starts with an empty energy queue at time 0.

A sampling policy or sensing scheduling policy is denoted as $\{l_n\}_{n=1}^{\infty}$, where l_n is the n-th sensing time instant. Let $l_0 = 0$. Let $d_n := l_n - l_{n-1}$, $n = 1, 2, \ldots$. Define $A(d_n)$ as the total amount of energy harvested in $[l_{n-1}, l_n)$, and $E(l_n^-)$ as the energy level of the sensor right before the scheduled sensing epoch l_n. Then, under any feasible sensing scheduling policy, the energy queue evolves as follows

$$E(l_{n+1}^-) = \min\{E(l_n^-) - 1 + A(d_{n+1}), B\} \tag{1}$$

$$E(l_n^-) \geq 1 \tag{2}$$

for $n = 1, 2, \ldots$. Based on the Poisson arrival process assumption, $A(d_{n+1})$s are independent Poisson random variables with parameters d_{n+1}s.

B. Sensing Performance Metric

We assume the sensing performance depends on how the sensing epochs are placed in time. Given that the durations between two consecutive sensing epochs are d_n, $n = 1, 2, \ldots$, the sensing performance over the sensing period is measured by $\sum_n f(d_n)$. We assume that function $f(d)$ has the following properties:

1) $f(d)$ is convex and monotonically increasing in d.
2) $f(d)/d$ is increasing in d.
3) $f(d) \leq Cd$, where C is a positive constant.

One example application that fits this model is to use samples collected at discrete time instances to estimate a time evolving physical quantity (temperature, humidity, etc), which is modeled as a random process with power-law decaying covariance. For such special random processes, it is shown that the linear minimum MSE (MMSE) estimation for any point on the random process only requires the two adjacent discrete-time samples bounding the point. In this case, $f(d)$ can be interpreted as the total mean square error over the length d interval bounded by two consecutive sensing epochs [4]. Optimizing the overall sensing performance is equivalent to minimizing the total mean square error of the linear MMSE over the whole sensing period.

For a clear exposition of the result, we assume that two samples at time 0 and time T are available at the sensor for free, i.e., no energy is used for collecting those two samples. Denote these two sampling epochs as $l_0 = 0$, $l_{N_T+1} = T$. Besides, there are N_T sensing epochs placed over $(0, T)$. The overall sensing performance over the duration $[0, T]$ is then a summation of $f(d_n)$, $n = 1, 2, \ldots, N_T + 1$.

C. Problem Formulation

Our objective is to design an online sensing policy $\{l_n\}$, such that the expected long-term average sensing utility is optimized, subject to the energy constraint at every time instant. The optimization problem is formulated as

$$\begin{aligned} \min_{\{l_n\}} \quad & \limsup_{T \to +\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\ \text{s.t.} \quad & (1)-(2) \end{aligned} \tag{3}$$

where the expectation in the objective function is taken over all possible energy harvesting sample paths. This is a stochastic optimization problem, and in general is hard to obtain a closed-form optimal solution.

III. Sensing Scheduling with Infinite Battery

In [4], we studied the optimal sensing scheduling policy for a similar energy harvesting sensing system under *infinite* battery constraint. We showed that the sensing performance (i.e., time-average MSE) has a lower bound, which can be achieved by a best-effort sensing scheduling policy. For the sake of completeness, we adapt and include the lower bound and the corresponding sensing scheduling policy below without providing the proofs.

Lemma 1 *The objective function in (3) is lower bounded as*

$$\min_{\{l_n\}} \limsup_{T \to +\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \geq f(1) \tag{4}$$

Definition 1 (Best-effort Sensing Scheduling) *The sensor is scheduled to perform the sensing task at $s_n = n$, $n = 1, 2, \ldots$. The sensor performs the sensing task at s_n if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled sensing epoch.*

Here we use s_n to denote the n-th *scheduled* sensing epoch, which is in general different from the *actual* sensing epoch l_n since some of the scheduled sensing epochs may be infeasible.

Theorem 1 *The best-effort sensing scheduling policy is optimal when the battery size is infinite, i.e.,*

$$\liminf_{T \to +\infty} \frac{1}{T} \sum_{n=1}^{N_T+1} f(d_n) = f(1) \quad a.s.$$

where d_n is the duration between the actual sensing epochs l_n and l_{n-1}.

Theorem 1 indicates that for almost every energy harvesting sample path, the best-effort sampling policy converges to the lower bound in Lemma 1 when the battery size is infinite. This is due to the fact that when the battery size is infinite, the fluctuations of the energy arrivals can be averaged out when time is sufficiently large, thus a uniform sensing scheme with sensing rate equal to the energy harvesting rate is optimal. However, with finite battery, the best-effort sampling scheme may not achieve the lower bound, since energy overflow is inevitable in this situation, which in turn results in more frequent infeasible sensing epochs due to battery outage.

IV. Sensing Scheduling with Finite Battery

In order to optimize the sensing performance when the battery size is finite, intuitively, the sensing policy should try to prevent any battery overflow, as wasted energy leads to performance degradation. Meanwhile, the properties of the sensing performance function requires the sensing epochs to be as uniform as possible. Those two objectives are not aligned with each other, thus, the optimal scheduling policy should strike a balance between them.

In the following, we propose an energy-aware adaptive sensing scheme. Different from the best-effort sensing scheduling policy which schedules the sensing epochs uniformly, the proposed sensing policy adaptively changes its sensing rate based on the instantaneous battery level. Intuitively, when the battery level is high, the sensor should sense more frequently in order to prevent battery overflow; When the battery level is low, the sensor should sense less frequently to avoid infeasible sensing epochs. Meanwhile, the sensing rate should not vary significantly so that a relatively uniform sensing scheduling can be achieved.

Definition 2 (Energy-aware Adaptive Sensing Scheduling) *Define $\beta := \frac{k \log B}{B}$ where k is a positive number such that $0 < \beta < 1$. The adaptive sensing scheduling policy defines sensing epochs s_n recursively as follows*

$$s_n = s_{n-1} + \begin{cases} \frac{1}{1-\beta} & E(s_{n-1}^-) < \frac{B}{2} \\ 1 & E(s_{n-1}^-) = \frac{B}{2} \\ \frac{1}{1+\beta} & E(s_{n-1}^-) > \frac{B}{2} \end{cases} \tag{5}$$

where $s_0 = 0$, $E(s_0^-) = 1$. The sensor performs the sensing task at s_n if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled sensing epoch.

The policy divides the battery state space into three different regimes. At each scheduled sensing epoch, the sensor decides whether to sense based on its current battery state, and adaptively selects the next sensing epoch depending on which regime the current battery state falls in. When it is above $B/2$, the sensor senses every $\frac{1}{1+\beta}$ units of time, and when it is below $B/2$, it senses every $\frac{1}{1-\beta}$ units of time. The value of β controls the deviation of the sensing rates. Intuitively, when the value of β increases, the probability that the battery overflows decreases, so does the probability that a scheduled sensing epoch is infeasible. However, larger β may also lead to larger deviations of the durations between sensing epochs, which results in sensing performance degradation.

The performances of the adaptive sensing scheme is characterized analytically in the following two theorems.

Theorem 2 *Under the adaptive sensing scheduling policy, the probability that a scheduled sensing epoch is infeasible scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$, and the average amount of wasted energy per unit time scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$.*

Theorem 2 indicates that when B is sufficiently large, both upper bounds of the battery outage probability and the overflow probability decrease monotonically as k increase. As the battery size B increases, the upper bounds of those two probabilities decrease and eventually approaches zero.

Theorem 3 *Under the adaptive sensing scheduling policy, the gap between the time average sensing performance and the lower bound $f(1)$ scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$.*

Theorem 3 implies that as battery size B increases, the sensing performance under the adaptive sensing scheduling policy approaches the lower bound achievable for the system with infinite battery. Thus, it is asymptotically optimal. Compared to the bounds in Theorem 2, the bound in Theorem 3 has an extra term $\left(\frac{\log B}{B}\right)^2$. For a sufficiently large B, the bound is dominated by the first term when k is small, and it is dominated by the second term when k is large. Thus, it may not monotonically decrease as k increase, which is consistent with the fact that the sensing performance is not only related to the battery outage and overflow probabilities, but also depends on the durations between consecutive sensing epochs.

The proofs of Theorems 2 and 3 are provided in the Appendix. The sketch of the proof is as follows. The battery states at scheduled sensing epochs form a discrete-time random process $\{E(s_n^-)\}_{n=1}^{\infty}$. However, it differs from a conventional discrete-time random process since the durations between two consecutive time indices vary in time: it could be $\frac{1}{1-\beta}$, $\frac{1}{1-\beta}$ or 1 depending on the battery state. This makes the analysis very complicated. To simplify the analysis, we consider the portion of $\{E(s_n^-)\}_{n=1}^{\infty}$ lying in $(0, B/2]$ and $[B/2, B)$ separately. The portion lying in each region can be mapped to a random process with uniformly spaced time indices. We characterize the expected time durations that the random process hits its boundaries for the first time, whose inverse indicates the corresponding battery overflow or outage rates. Theorem 3 can be proved based on such characterization and properties of the sensing performance function $f(d)$.

V. Simulation Results

We evaluate the performance of the proposed sampling policy through simulations. Fixing the energy harvesting rate to be $\lambda = 1$ unit energy per unit time, and $T = 100,000$, we generate a sample path for the Poisson energy harvesting process, and perform the proposed adaptive sensing scheduling policy. We adopt the MSE function for random process reconstruction in [4] to measure the sensing performance under the proposed sensing scheme. Specifically, the correlation between two samples separated by a time duration d is ρ^d, and the average reconstruction MSE of the random field between two d-spaced samples is

$$f(d) = d\frac{1+\rho^{2d}}{1-\rho^{2d}} + \frac{1}{\log \rho} \tag{6}$$

In the simulation, we use $\rho = 0.7$.

We keep track of the following quantities. First, we count the total number of sensing epochs under the policy, denoted as N_T. Among those N_T sensing epochs, we count the total number of infeasible ones (i.e., the epoch s_n when $E(s_n^-) < 1$), and divide it by N_T. This gives us the ratio of infeasible sensing epochs to all scheduled sensing epochs under the policy. We let $k = 0, 1, 2$, respectively, and perform the adaptive sensing according to (5) with battery size B varying from 2 to 100. The corresponding results are plotted in Fig. 1. We note that for each fixed k, the ratio monotonically decreases as B increase, and each curve is roughly convex in B. This is consistent with the theoretical bounds in Theorem 2. Meanwhile, for each fixed battery size, the ratio decreases as k increases. This is due to the fact that the adaptive sensing policy is more conservative for larger k when battery level is below $B/2$, i.e., it senses at a slower rate for larger k, which makes the energy level drift away from empty state with higher probability.

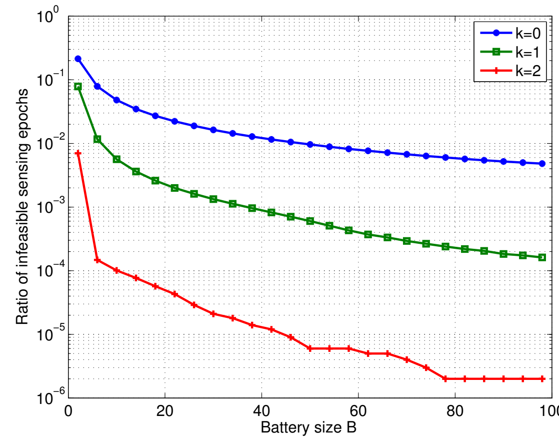

Fig. 1: The ratio of infeasible sensing epochs.

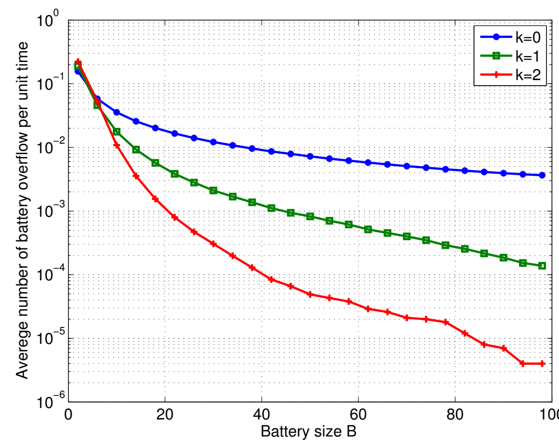

Fig. 2: The average number of battery overflow per unit time.

Next, we study battery overflow under the proposed policy. We count the total number of time instants when the battery state exceeds B, and divide it by T. The average number of battery overflow events per unit time is plotted as a function of B in Fig. 2 for $k = 0, 1, 2$, respectively. Again, we observe that for each fixed k, the curve is monotonically decreasing and roughly convex in B, as predicted by the theoretical bounds in Theorem 2. Meanwhile, for each fixed battery size, the battery overflow rate decreases as k increases. This is due to the fact that the adaptive sensing policy is more aggressive for larger k when battery level is above $B/2$, i.e., it senses at a faster rate for larger k. Thus, the energy level drifts away from full state with higher probability.

At last, we study the sensing performance in terms of the time-average MSE. We calculate the MSE for each interval bounded by two consecutive sensing epochs as (6), aggregate them and divide the sum by T. The time-average MSE is plotted in Fig. 3. We note that for each fixed k, the gap between the time-average MSE and the lower bound monotonically decreases as B increases, which is consistent with the theoretical result in Theorem 3. However, when B is fixed, the best sensing performance is observed at $k = 1$, which is different from the results in Fig. 1 and Fig. 2. Even though the battery outage and overflow rates decrease in k, the average sensing performance does not exhibit such monotonicity. This is because when k is large, the sensing rate varies dramatically in time. Although this leads to lower outage and overflow probabilities, it compromises the sensing performance as the sensing scheduling deviates from the desired uniform sensing scheduling. Thus, there exists a tradeoff between reducing battery outage and overflow probabilities, and equalizing the sensing rates in time. The optimal selection of k should jointly consider those two conflicting objectives.

VI. Conclusions

In this paper, we considered the optimal online sensing scheduling policy for an energy harvesting sensing system with a finite battery. We first provided a lower bound on the long-term average sensing performance for the system without

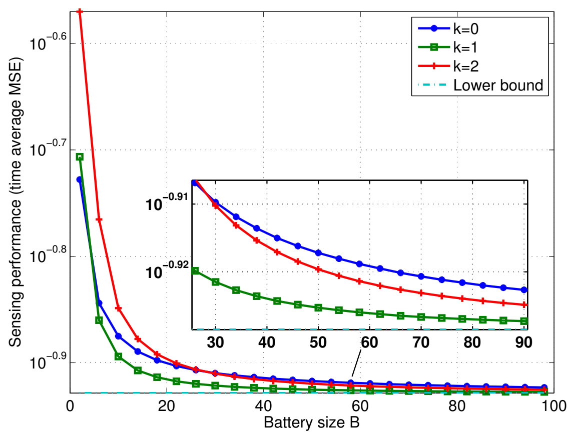

Fig. 3: The time-average sensing MSE.

the finite battery constraint. We then proposed an energy-aware adaptive sensing scheduling policy, which dynamically varies the sensing rate based on instantaneous energy levels of the battery. We showed that the battery outage and overflow probabilities under the proposed policy approach zero as battery size goes to infinity, and the long-term average sensing performance converges to the lower bound. Thus the adaptive sensing scheduling policy is asymptotically optimal. The convergence rates as a function of the battery size were also explicitly characterized. Simulation results validated the theoretical bounds.

APPENDIX

The proofs to Theorems 2 and 3 are provided here. Our approach can be sketched as follows: In Appendix A, we construct a "virtual" energy harvesting sensing system, whose battery state can be any integer in $(-\infty, +\infty)$. Assuming the virtual sensing system senses at a uniform rate, we analytically characterize the expected duration between two consecutive events that the virtual battery state hits a certain level. In Appendix B, we show that how the original sensing system with a finite battery can be mapped to the virtual system, and exploit the analytical results in Appendix A to prove Theorem 2. In Appendix C, we use the results from Appendix B to prove Theorem 3.

A. A Virtual Energy Harvesting Sensing System

Consider an energy harvesting sensing system with a virtual battery whose state can be any integer in $(-\infty, +\infty)$. It senses every $\frac{1}{1-\beta}$ units of time, even if the battery state is zero or negative. The energy arrives at the virtual battery according to a Poisson process with parameter 1. Each sensing operation consumes one unit of energy. We use $E_\beta(n)$ to denote the battery state right before the n-th sensing epoch, i.e., at time $\frac{n}{1-\beta}$. Assume the system starts with initial energy level x, then, the battery status evolves according to

$$E_\beta(0) = x \tag{7}$$

$$E_\beta(n) = E_\beta(n-1) + A\left(\frac{1}{1-\beta}\right) - 1, \quad n = 1, 2, \ldots \tag{8}$$

where $A\left(\frac{1}{1-\beta}\right)$ is a Poisson random variable with parameter $\frac{1}{1-\beta}$. Thus,

$$\mathbb{E}[E_\beta(n)] = x + \frac{\beta}{1-\beta}n \tag{9}$$

Therefore, when $0 < \beta < 1$, the energy level drifts up in expectation; Otherwise, when $\beta < 0$, it drifts down.

Define

$$\Lambda_\beta(\alpha) := \log \mathbb{E}\left[e^{-\alpha\left(A\left(\frac{1}{1-\beta}\right)-1\right)}\right] = \frac{e^{-\alpha}-1}{1-\beta} + \alpha \tag{10}$$

We note that $\Lambda_\beta(\alpha)$ is convex in α, $\Lambda_\beta(0) = 0$, and $\Lambda'_\beta(\alpha) = -\frac{e^{-\alpha}}{1-\beta} + 1$. Thus, equation $\Lambda_\beta(\alpha) = 0$ has another root besides 0, denoted as α_0. We have

$$\frac{e^{-\alpha_0}-1}{1-\beta} + \alpha_0 = 0, \quad \Lambda'_\beta(0) = -\frac{\beta}{1-\beta} \tag{11}$$

When α_0 is sufficiently small, we have $\beta = \frac{\alpha_0}{2} + o(\alpha_0)$.

Assume $x \in (0, M)$, where M is a positive integer. We are interested in the event that the random process $\{E_\beta(n)\}_{n=0}^\infty$ hits or exceeds one of the two boundary levels 0 and M for the first time. We have the following observations.

Lemma 2 *Consider the random process $\{E_\beta(n)\}_{n=0}^\infty$ defined in (7)-(8). Let κ be the smallest n such that $E_\beta(n) \geq M$ or $E_\beta(n) = 0$, and $\tau_x := \mathbb{E}[\kappa]$. Define $P_{x,M}$ as the probability that $E_\beta(\kappa) \geq M$, and $P_{x,0}$ as the probability that $E_\beta(\kappa) = 0$. Then,*

$$P_{x,M} = \frac{1-e^{-\alpha_0 x}}{1-e^{-\alpha_0(M+\theta_x)}} \tag{12}$$

$$P_{x,0} = \frac{e^{-\alpha_0 x} - e^{-\alpha_0(M+\theta_x)}}{1-e^{-\alpha_0(M+\theta_x)}} \tag{13}$$

$$\tau_x = \frac{1-\beta}{\beta}((M+\phi_x)P_{x,M} - x) \tag{14}$$

where $0 \leq \theta_x \leq \frac{1}{1-\beta}$, $0 \leq \phi_x \leq \frac{1}{1-\beta}$.

Lemma 3 *Consider the random process $\{E_\beta(n)\}_{n=0}^\infty$ defined in (7)-(8). Define $S^-_{x,M}$ as the expected time index n when $\{E_\beta(n)\}_{n=0}^\infty$ with $\alpha_0 = -\frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) < 0$ hits boundary level M for the first time, and $S^+_{x,0}$ as the expected time index n when $\{E_\beta(n)\}_{n=0}^\infty$ with $\alpha_0 = \frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) > 0$ hits boundary level M for the first time. Then, $S^-_{M,M} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$, $S^+_{0,0} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$.*

The proofs of Lemmas 2 and 3 are omitted due to space limitations.

B. Proof of Theorem 2

Now consider the energy state evolution process $\{E(s_n^-)\}_{n=1}^{\infty}$ under the proposed adaptive sensing scheduling policy. We focus on the portion of the random process lying in range $[0, B/2)$ and $(B/2, B]$, respectively. Comparing the random process $\{E(s_n^-)\}_{n=1}^{\infty}$ with the virtual battery evolution process defined in (7)-(8), we note that each portion can be treated as part of $\{E_\beta(n)\}_{n=0}^{\infty}$ lying in the corresponding range. Therefore, the characterization of $\{E_\beta(n)\}_{n=0}^{\infty}$ in Lemma 2 and Lemma 3 can be slightly modified to characterize $\{E(s_n^-)\}_{n=1}^{\infty}$.

Specifically, for the portion lying in $[0, B/2)$, we let $M = B/2$, $\beta = \frac{k \log B}{B} > 0$, then, the expected number of epochs between two consecutive battery outage events, i.e., $E(s_n^-) = 0$, can be bounded below by $S_{0,0}^+$. Thus, based on law of large numbers, the probability that a sensing epoch is infeasible is bounded above by $1/S_{0,0}^+$. Therefore, it scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$.

Similarly, for the portion lying in $[B/2, B)$, we map $B \to M$, $B/2 \to 0$, $\beta = -\frac{k \log B}{B} < 0$, then, the expected number of epochs between two consecutive battery overflow events, i.e., $E(s_n^-) = B$, can be bounded below by $S_{M,M}^-$. Again, based on law of large numbers, the rate of battery overflow scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$. Due to the properties of Poisson process, we can show that the amount of wasted energy per unit time is bounded by twice of the battery overflow rate, thus it scales in the same order.

C. Proof of Theorem 3

Consider the first n scheduled sensing epochs under the proposed adaptive sensing scheduling policy. Let n_+ denote the number of intervals between two scheduled sensing epochs with duration $\frac{1}{1-\beta}$, n_- be that with duration $\frac{1}{1+\beta}$, and n_0 be that with duration 1. Let $\bar{n}$ be the number of sensing epochs the battery overflows, and $\underline{n}$ be the number of infeasible sensing epochs. Then, the n-th scheduled sensing epoch happens at time $T_n := \frac{n_+}{1-\beta} + n_0 + \frac{n_-}{1+\beta}$. Let A_n^+ be the total amount of energy wasted. Then,

$$E(S_n^-) = (A(T_n) - A_n^+) - (n - \underline{n}) \tag{15}$$

where $A(T_n)$ is a Poisson random variable with parameter T_n. Dividing both sides by n and taking the limit as n goes to $+\infty$, we have

$$\lim_{n\to\infty} \frac{E(n)}{n} = \lim_{n\to\infty} \frac{A(T_n)}{T_n} \cdot \frac{T_n}{n} - \lim_{n\to\infty} \frac{A_n^+}{n} - \left(1 - \lim_{n\to\infty} \frac{\underline{n}}{n}\right)$$

Therefore,

$$\lim_{n\to\infty} \frac{T_n}{n} = 1 + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{16}$$

Based on Taylor expansion and (16), we have

$$\lim_{n\to\infty} \frac{n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)}{T_n} = f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$$

On the other hand, due to the existence of infeasible sensing epochs, we have

$$\lim_{n\to\infty} \frac{\sum_n f(d_n) - \left[n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)\right]}{T_n}$$

$$\le \lim_{n\to\infty} \frac{\sum_{d_n: d_n \ge \frac{1}{1-\beta}} f(d_n)}{T_n} \tag{17}$$

$$\le \lim_{n\to\infty} \frac{\sum_{d_n: d_n \ge \frac{1}{1-\beta}} C d_n}{T_n} \tag{18}$$

$$\le \lim_{n\to\infty} \frac{2C\underline{n}}{T_n} = O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{19}$$

where (17) follows from the fact that the difference between the actual sensing performance and scheduled sensing performance is due to the infeasible sensing epochs. (18) follows from the property of $f(d)$, and (19) follows from Theorem 2 and (16). Thus,

$$\lim_{n\to\infty} \frac{\sum_n f(d_n)}{T_n} = f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$$

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