Optimum Sensing of a Time-Varying Random Event with Energy Harvesting Power Sources

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***Abstract*—In this paper, we study the optimum estimation of a continuous-time random process by using discrete-time samples taken by a sensor powered by energy harvesting power sources. The system employs a best-effort sensing scheme to cope with the stochastic nature of the energy harvesting sources. The best-effort sensing scheme defines a set of equally-spaced candidate sensing instants, and the sensor performs sensing at a given candidate sensing instant if there is sufficient energy available, and remains silent otherwise. It is shown through asymptotic analysis that when the energy harvesting rate is strictly less than the energy consumption rate, there is a non-negligible percentage of silent symbols due to energy outage. For a given average energy harvesting rate, a larger sampling period means a smaller energy outage probability and/or more energy per sample, but a weaker temporal correlation between two adjacent samples. Such a tradeoff relationship is captured by developing a closed-form expression of the estimation MSE, which analytically identifies the interactions among the various system parameters, such as the ratio between the energy harvesting rate and energy consumption rate, the sampling period, and the energy allocation between sensing and transmission. It is shown through theoretical analysis that the optimum performance can be achieved by adjusting the sampling period and sampling energy such that the average energy harvesting rate is equal to the average consumption rate.**

***Index Terms*—Energy harvesting, stochastic energy sources, optimum sampling, MSE**

I. Introduction

In a wireless sensing system, energy is consumed during the operations of both sensing and information transmission. Many wireless sensors are expected to operate autonomously over an extended period of time under extremely stringent energy constraints, and this necessitates the design of sensors powered by devices that can harvest energy from the ambient environment. However, the stochastic nature of the harvested energy imposes formidable challenges on the designs of systems with energy harvesting power sources.

The designs of energy harvesting sensing and communication systems have attracted considerable interests. Many existing works employ off-line scheduling methods, which identify the optimum transmission scheduling based on full knowledge of current and future energy arrivals [9], [1]. The off-line scheduling methods are non-causal and thus cannot be applied to practical systems. On-line scheduling methods, on the other hand, use only the statistics of energy arrivals, and they can be formulated as stochastic dynamic programming problem with high complexity [4]. Low complexity sub-optimum on-line algorithms are presented in [4] and [6]. In both works, the performance of all on-line scheduling policies is strictly worse than that of the off-line scheduling.

Recently it has been shown that there is an asymptotic equivalence between systems with stochastic and deterministic energy sources, if and only if the average energy harvesting rate is no less than the average energy consumption rate in systems with infinite battery capacity [8], [10], [12]. Therefore, on-line scheduling with stochastic energy sources can asymptotically achieve the same performance as off-line scheduling. Specifically, it is shown in [8], [10] that the asymptotic equivalence can be achieved with an on-line best-effort sensing policy, where a sensing is performed at equally-spaced candidate sensing instants whenever there is sufficient energy to do so, and the sensor will remain silent otherwise. Best-effort sensing with finite battery is studied in [11]. The above works only consider the case when the ratio between the average energy harvesting rate and consumption rate is no less than one. The design and performance of systems with the energy harvesting-consumption ratio less than one remains an open problem.

In this paper, we study the optimum designs of energy harvesting sensing system when the energy harvesting rate is no more than the energy consumption rate. The system tries to reconstruct a time-varying wide-sense stationary (WSS) random event by using discrete samples. The optimum sampling of a random field is a classical problem and has been studied extensively in the literature [5], [3]. It is shown that uniform sampling can achieve the optimum performance for a wide range of kernel covariance functions [3], [7]. However, uniform sampling is in general infeasible in energy harvesting systems due to possible energy outage. To cope with stochastic energy sources, we adopt the best-effort sensing policy [8].

When the energy harvesting-consumption rate is less than one, it is shown through asymptotic analysis that there is a non-negligible probability of energy outage at candidate sensing instants, and the outages will significantly degrade system performance. Intuitively, for a given average energy harvesting rate, we can reduce the energy outage probability by either increasing the sampling period or reducing the energy allocated for each sample. However, a larger sampling period means weaker sample correlation; lower energy per sample results in a lower signal-to-noise ratio (SNR). Thus it is critical to strike a balance among energy outage, sample correlation, and SNR

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to achieve the optimum performance. We capture the tradeoff relationship by developing a closed-form expression of the estimation MSE, which analytically identifies the interactions and impacts of various system parameters, such as the energy harvesting-consumption ratio, the sampling period, and the ratio between energy allocated to sensing and transmission. It is shown analytically that the optimum performance is achieved when the energy harvesting-consumption ratio is one, and this can be achieved by adjusting the sampling period and energy per sample under a given energy harvesting rate. On the other hand, the performance degrades when the energy harvesting-consumption ratio is less than one, and the rate of degradation depends on the sampling period.

II. System Model

Consider a sensor used to monitor a WSS time-varying random event. The sensor is powered by an energy harvesting device. The harvested energy can be modeled as a random process. We have two assumptions regarding the stochastic energy model:

Assumption 1: If we divide the time axis into arbitrary small intervals with length $\Delta > 0$, then the energy collected in each interval can be modeled as independently and identically distributed (i.i.d.) random variables, E_Δ, with mean $P\Delta$, where P is the average harvesting power.

Assumption 2: $\sum_{n=1}^{\infty} P(E_\Delta > n\epsilon) < \infty$ for any $\epsilon > 0$ and $\Delta > 0$.

Such a model is general enough to incorporate many other existing stochastic energy models, such as Poisson energy source [4] and Bernoulli energy source [2], as special cases.

The harvested energy is stored in an energy storage device, such as rechargeable batteries or super capacitors. Since the harvested energy is usually very small compared to the capacity of the energy storage device, it is assumed that the energy queue has unlimited capacity [8], [10], [12]. Denote the amount of energy available in the energy storage device at time t as $Q(t) \geq 0$. The energy consumption must follow the energy causality constraint, that is, at any time instant, the total amount of harvested energy must be no less than the total amount of consumed energy.

The time-varying event being monitored is modeled as a WSS random process $s(t)$, where t is the time variable. It is assumed that $s(t)$ is zero mean with a covariance function $R_{ss}(t) = R_{ss}(t_2 - t_1) = \mathbb{E}[s(t_1)s(t_2)] = \rho^{|t_2-t_1|}$, where $\rho \in [0, 1]$ is the power-law scaling coefficient, and $\mathbb{E}$ is the mathematical expectation operator. The power law covariance function is an alternative representation of the Ornstein-Uhlenbeck covariance kernel [5].

The sensor attempts to reconstruct the continuous-time time-varying random event by using noise-distorted discrete-time observations of the random process. A sensing policy is defined as a sequence of time instants $\{t_n\}_n$, where t_n is the time instant at which the sensor collects a sample of the random process.

At a given time instant, the sensor collects a sample, and then transmits it to a fusion center (FC), which tries to reconstruct the continuous-time random event by using the collection of discrete-time samples. Assume the sensing and transmission of one sample consumes E_0 joules of energy, where αE_0 is used for sensing and $(1-\alpha)E_0$ for transmission, with $\alpha \in (0, 1)$ being the energy allocation factor.

The sample collected by the sensor at time t_n is

$$x(t_n) = \sqrt{\alpha E_0}\, s(t_n) + w(t_n) \tag{1}$$

where $w(t_n)$ is the sensing noise with a zero-mean and the auto-covariance function $\mathbb{E}\left[w(t_1)w(t_2)\right] = \sigma_w^2 \delta(t_1 - t_2)$, and $\delta(t)$ is the Dirac delta function. The sample $x(t_n)$ is first normalized to unit energy by multiplying with $\frac{1}{\sqrt{\alpha E_0 + \sigma_w^2}}$. The sample is then transmitted to the FC with energy $(1-\alpha)E_0$, and the signal observed at the FC is

$$y(t_n) = \sqrt{\frac{(1-\alpha)E_0}{\alpha E_0 + \sigma_w^2}}\, x(t_n) + v(t_n), \tag{2}$$

where $v(t)$ is the channel noise with a zero-mean and variance σ_v^2. It should be noted that the sensing and channel noise components are not necessarily Gaussian distributed.

The sensing system attempts to reconstruct the time-varying random field, $s(t)$, by using the sequence of the discrete-time samples, $\{y(t_n)\}_n$.

III. Statistical Properties of Energy Source

In this section, we study the asymptotic behaviors of the best-effort sensing policy as time goes to infinity.

The best-effort sensing policy [8] is adopted to cope with the stochastic nature of the energy source. For completeness, the best-effort sensing policy is defined as follows.

Definition 1 (best-effort Sensing Policy). *Define a set of candidate sensing instants as $\mathcal{K} = \{k_n | k_n = nT_s, n = 1, 2, \cdots\}$. A sensor performs one sensing operation with energy E_0 at time t if and only if: 1) $t \in \mathcal{K}$, and 2) $Q(t) \geq E_0$.*

In the best-effort sensing policy, the sensor attempts to mimic uniform sampling with its best efforts. It tries to perform sensing operations at uniform sensing intervals whenever allowed by the energy constraint. However, it will keep silent at a candidate sensing instant nT_s if $Q(nT_s) < E_0$. Denote the information collected at each candidate sensing instant as a sensing symbol, which could be either a *silent symbol* when $Q(nT_s) < E_0$ or an *active symbol* when $Q(nT_s) \geq E_0$. With such a sensing mechanism and the stochastic energy source, there might be K silent symbols in the first $N \geq K$ sensing instants $T_s, 2T_s, \cdots, NT_s$. The number of silent symbols is a random variable. The existence of silent symbols might degrade the sensing performance.

Definition 2. *Based on the best-effort sensing policy in Definition 1, the energy harvesting-consumption ratio is defined as*

$$q = \frac{PT_s}{E_0} \tag{3}$$

where P is the average energy harvesting rate.

It has been shown in [8] that the best-effort sensing policy can asymptotically achieve the same performance as uniform sensing, if and only if the average energy harvesting rate is no less than the average energy consumption rate, that is, $q \geq 1$.

In this paper, we are interested in the case that $q \leq 1$. When $q \leq 1$, it is expected that the performance of systems with stochastic energy sources will be worse than their counterparts with deterministic energy sources. The performance loss will be quantified through asymptotic analysis.

Theorem 1. *Consider an energy harvesting sensing system with energy source satisfying Assumptions 1 and 2, and it employs the best-effort sensing policy described in Definition 1. Define $K = \sum_{k=1}^{N} \mathbf{1}_{Q(kT_s)<E_0}$ as the total number of silent symbols in the first N symbol periods, where the indicator function $\mathbf{1}_{\mathcal{E}} = 1$ if the event $\mathcal{E}$ is true and 0 otherwise. If the energy harvesting-consumption ratio satisfies $q \leq 1$.*

$$\lim_{N\to\infty} \frac{K}{N} = 1 - q, \quad a.s. \tag{4}$$

where $q = \frac{PT_s}{E_0}$ is the energy harvesting-consumption ratio.

The results in Theorem 1 indicates that, as time goes to infinity, the probability of silent sensing symbol due to energy outage is equal to one minus the energy harvesting-consumption ratio q. Therefore, as $q < 1$, there is a non-diminishing number of silent symbols, and they might significantly degrade the system performance. The results hold for a quite general category of energy harvesting processes.

IV. Asymptotically Optimum Sensing with the Best-Effort Sensing Policy

This section studies the optimum design and performance analysis of sensing systems employing the best-effort sensing policy with the energy harvesting-consumption ratio $q \leq 1$.

A. Optimum Receiver

We can use a sequence of indicator variables, λ_n, to distinguish between active and silent sensing symbols. That is, $\lambda_n = 1$ if a sample is collected and transmitted at nT_s, and $\lambda_n = 0$ otherwise. Based on the results in Theorem 1, as n becomes large enough and $q \leq 1$, λ_n can be modeled as i.i.d. RVs with $\Pr(\lambda_n = 1) = q$ and $\Pr(\lambda_n = 0) = 1 - q$. The following analysis is based on the assumption that n is large enough such that the system enters steady state.

With the best-effort sensing policy, the sensing sample at time instant nT_s can be represented as $\lambda_n s_n$, where $s_n = s(nT_s)$. From (1) and (2), the signal observed by the FC at nT_s is

$$\begin{aligned} y_n &= \sqrt{\frac{\alpha(1-\alpha)}{\alpha E_0 + \sigma_w^2}} E_0 \lambda_n s_n + \sqrt{\frac{(1-\alpha)E_0}{\alpha E_0 + \sigma_w^2}} \lambda_n w_n + v_n, \quad (5) \\ &= A\lambda_n s_n + z_n \quad (6) \end{aligned}$$

where $w_n = w(nT_s)$, $v_n = v(nT_s)$, $A = \sqrt{\frac{\alpha(1-\alpha)}{\alpha E_0 + \sigma_w^2}} E_0$, and $z_n = \sqrt{\frac{(1-\alpha)E_0}{\alpha E_0 + \sigma_w^2}} \lambda_n w_n + v_n$. Since the FC does not know which symbol is silent or active, it will still observe the noise z_n during a silent symbol. The variance of z_n is

$$\sigma_z^2 = \frac{(1-\alpha)E_0}{\alpha E_0 + \sigma_w^2} q\sigma_w^2 + \sigma_v^2. \tag{7}$$

The FC tries to reconstruct $s(t)$ by using $\{y_n\}_n$. Here we consider the worst case scenario by estimating $\{d_n \triangleq s\left(nT_s + \frac{1}{2}T_s\right)\}_n$, the sequence of points located in the middle between two candidate sensing instants. It should be noted that $s\left(nT_s + \frac{1}{2}T_s\right)$ will be estimated even if $\lambda_n = 0$ and/or $\lambda_{n+1} = 0$.

The linear minimum mean squared error (MMSE) estimation of d_n based on $\{y_n\}_n$ is

$$\hat{d}_n = \sum_{k=-\infty}^{\infty} h_k y_{n-k}, \tag{8}$$

where $\{h_k\}_k$ is the impulse response of the MMSE filter.

Based on the orthogonal principle, $\mathbb{E}\left[(\hat{d}_n - d_n)y_m\right] = 0$, we have

$$\sum_{k=-\infty}^{\infty} h_k r_{yy}(n-k) = r_{dy}(n) \tag{9}$$

where $r_{yy}(n) = \mathbb{E}[y(m+n)y(m)]$ and $r_{dy}(n) = \mathbb{E}[d(m+n)y(m)]$.

From (6), we have

$$r_{dy}(n) = Aqr_{ds}(n) \tag{10}$$

$$r_{yy}(n) = A^2q^2 r_{ss}(n)(1-\delta_n) + \left[A^2q + \sigma_z^2\right]\delta_n \tag{11}$$

where $r_{ds}(n) = \mathbb{E}[s(t+nT_s+\frac{1}{2}T_s)s(t)] = \rho^{|n+\frac{1}{2}|T_s}$, $r_{ss}(n) = \mathbb{E}[s(t+nT_s)s(t)] = \rho^{|n|T_s}$, $\delta_n = 1$ if $n = 0$ and $\delta_n = 0$ otherwise.

Converting (9)-(11) into the frequency domain with discrete-time Fourier transform (DTFT), we have

$$H(f) = \frac{AqR_{ds}(f)}{A^2q^2R_{ss}(f) + [A^2q(1-q) + \sigma_z^2]} \tag{12}$$

where $R_{ds}(f)$ and $R_{ss}(f)$ are the DTFTs of $r_{ds}(n)$ and $r_{ss}(n)$, respectively.

Based on (12), the filter coefficients $\{h_k\}_k$ can be obtained as the inverse DTFT of $H(f)$.

We have the following results regarding the mean squared error (MSE), $\sigma_e^2 = \mathbb{E}\left[(d_n - \hat{d}_n)^2\right]$.

Theorem 2. *Consider the sensing system defined in* (6)*. With the linear MMSE receiver given in* (9) *and* (12)*, the MSE for estimating d_n is*

$$\sigma_e^2 = \left(C + \frac{1-\rho^{T_s}}{1+\rho^{T_s}}\right)^{\frac{1}{2}} \left(C + \frac{1+\rho^{T_s}}{1-\rho^{T_s}}\right)^{-\frac{1}{2}}. \tag{13}$$

where

$$C = \frac{1}{q} - 1 + \frac{1}{\alpha\gamma_w T_s} + \frac{1}{q(1-\alpha)\gamma_v T_s} + \frac{1}{\alpha(1-\alpha)\gamma_v\gamma_w T_s^2} \tag{14}$$

with $\gamma_w = \frac{P}{\sigma_w^2}$, $\gamma_v = \frac{P}{\sigma_v^2}$, *and* $q = \frac{PT_s}{E_0} \in (0, 1]$.

In Theorem 2, the MSE is expressed as a function of three parameters: the energy allocation factor α, the energy harvesting-consumption ratio q, and the sampling period T_s. These three parameters feature the tradeoff among energy harvesting and consumption, and they can be optimized to minimize the MSE.

B. Optimum System Design

Define $g(x) = \frac{1+\rho^{T_s}}{1-\rho^{T_s}}$. The MSE given in (13) depends on both C and $g(T_s)$, and they play two opposite roles on the MSE. The MSE depends on q and α solely through C.

Corollary 1. *The MSE given in* (13) *is a decreasing function in* $q \in (0, 1]$.

The result in Corollary 1 indicates that given T_s and α, the MSE can be minimized by setting $q = 1$, that is, the energy harvesting-consumption ratio is 1 such that the probability of energy outage is 0. This can be achieved either by increasing the sampling period T_s, or reducing the energy per sample E_0, such that the average energy harvesting rate is no less than the average energy consumption rate in the best-effort sensing. It should be noted that increasing T_s or reducing E_0 might degrade system performance. Thus it is important to find the best trade-off among the various parameters.

On the other hand, when $q < 1$, the result in Theorem 2 quantifies the performance loss due to non-negligible energy outage caused by energy harvesting sources.

Corollary 2. *Given* T_s *and* q*, the energy allocation factor that minimizes the MSE given in Theorem 2 is*

$$\alpha^*(q, T_s) = \begin{cases} \frac{\sqrt{(1+\gamma_v T_s)(1+\gamma_w T_s/q)} - (1+\gamma_v T_s)}{\gamma_w T_s/q - \gamma_v T_s}, & \gamma_w \neq q\gamma_v \\ 0.5, & \gamma_w = q\gamma_v \end{cases} \tag{15}$$

From Corollary 1, the optimum q that minimizes σ_e^2 is 1, for all α and T_s. From Corollary 2, the optimum α that minimizes σ_e^2 is given in (15). Thus the optimum system can be achieved by setting $q = 1$ and $\alpha = \alpha^*(1, T_s)$ in (13), and then minimizing the equation with respect to T_s.

Setting $q = 1$ and $\alpha = \alpha^*(1, T_s)$ in (13) yields

$$\sigma_e^2(T_s) = \left[f(T_s) + g^{-1}(T_s)\right]^{\frac{1}{2}} \left[f(T_s) + g(T_s)\right]^{-\frac{1}{2}} \tag{16}$$

where $g(x) = \frac{1+\rho^x}{1-\rho^x}$ and

$$f(x) = \frac{1}{\alpha^*(1,x)\gamma_w x} + \frac{1}{[1-\alpha^*(1,x)]\gamma_v x} + \frac{1}{\alpha^*(1,x)[1-\alpha^*(1,x)]\gamma_v\gamma_w x^2} \tag{17}$$

The function $f(x)$ can be interpreted as the inverse of the effective SNR. Given P and $q = 1$, a larger T_s means a higher E_0, thus a higher SNR, which will decrease the MSE. The function $g(x)$ is determined by the correlation among samples. A larger T_s means a weaker sample correlation, which negatively contributes to the MSE performance.

Corollary 3. *The optimum* T_s *that minimizes the MSE is in the set* $T_s^* \in \{0\} \cup \{x | u(x) = 0\}$, *where*

$$u(x) = f'(x) - \log\rho - f(x)\log\rho\frac{1+\rho^{2x}}{1-\rho^{2x}} \tag{18}$$

The optimum sampling period can be solved with Corollary 3, and the optimum T_s^* can then be used in Corollary 2 to obtain the optimum energy allocation factor $\alpha^*(1, T_s^*)$. Our numerical results indicate that there is only one solution to $u(x) = 0$ for all configurations considered in this paper.

V. Numerical and Simulation Results

Fig. 1 shows the value of $\frac{K}{N}$ as a function of N for systems with Poisson or Bernoulli energy sources, where K is the number of silent sensing symbols and N is the total number of candidate sensing symbols. All systems employ the best-effort sensing policy. The simulation results are obtained by averaging over 100 independent runs for each configuration. Both energy sources have similar convergence behaviors as N increases. For all configurations, the values of $\frac{K}{N}$ converge to $1 - q$ as predicted in Theorem 1. The ratios converge to their respective limits as $N > 300$.

Fig. 2 shows the MSE as a function of the energy allocation factor under various system configurations. In all subsequent simulations, we have $\gamma_w = 10$ dB and $\gamma_v = 3$ dB. The power law scaling coefficient is $\rho = 0.9$. From the results, the MSE is convex in α. The optimum $\alpha^*(q, T_s)$ from Corollary 2 are marked as 'x' on the curves, and they match the values of α that minimize σ_e^2. Given α and T_s, the MSE decreases monotonically as q increases. In addition, it is apparent that the sampling period T_s has significant impact on the MSE performance. The performance of the system with $q = 0.4$ and $T_s = 0.5$ is better than that of the system with $q = 1$ and $T_s = 0.05$.

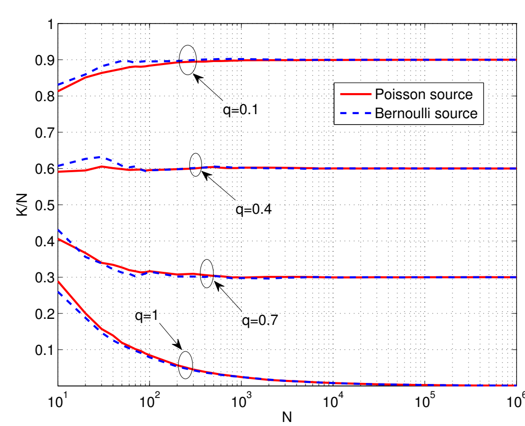

Fig. 1. The percentage of silent sensing symbols for various energy sources.

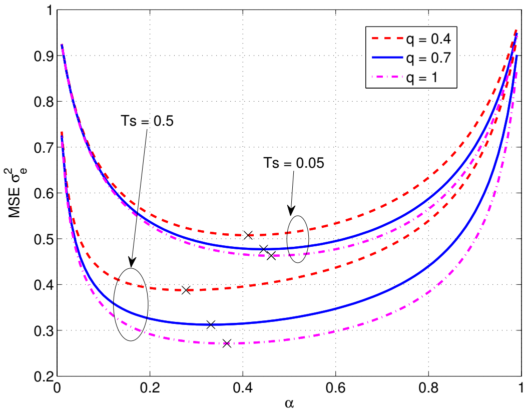

Fig. 2. The MSE as a function of the energy allocation factor α.

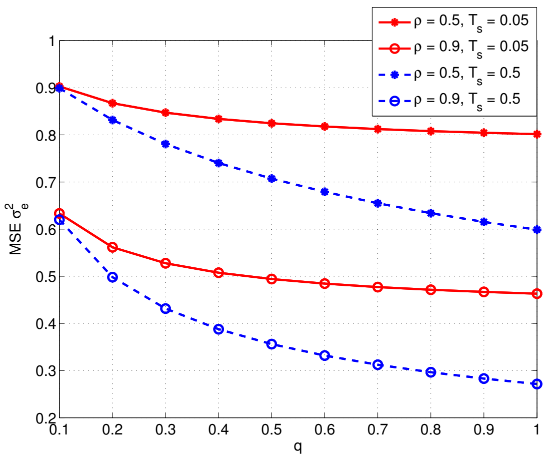

Fig. 3. The MSE as a function of the active symbol probability q under optimum energy allocation.

The impact of energy harvesting-consumption ratio, q, on the MSE is shown in Fig. 3. The energy allocation factors in all cases are the optimum values from Corollary 2. As predicted in Corollary 1, the MSE decreases monotonically with q, but with different slopes. For the same ρ, the T_s that renders a smaller MSE has a steeper slope. Therefore the gap between two systems with different values of T_s increases as q increases.

Fig. 4 demonstrates the impacts of T_s on σ_e^2, with the optimum $\alpha^*(q, T_s)$ and under different values of q. For all configurations, the MSE is quasi-convex in T_s with a single zero-slope point. The optimum T_s that minimizes the MSE is thus the unique solution of $u(x) = 0$ as shown in Corollary 3. As q increases, the optimum T_s^* becomes larger.

VI. Conclusions

This paper has studied the optimum sensing of a time-varying random event with energy harvesting power sources. It has been shown that the energy harvesting-consumption ratio plays a critical role on system performance, and the optimum performance is achieved when this ratio is one. When the energy harvesting-consumption ratio is strictly less than one,

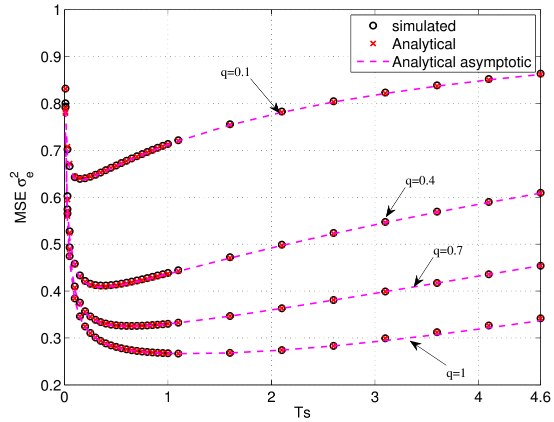

Fig. 4. The MSE as a function of the sampling period T_s under optimum energy allocation.

there is always a non-negligible probability of energy outage, which results in performance degradation. With asymptotic analysis, the estimation MSE has been expressed as a closed-form expression of several important design parameters, and the performance is optimized by striking a balanced tradeoff among the sampling period, the energy harvesting-consumption ratio, and the energy allocation factor. The results are general enough to include a wide range of energy harvesting sources.

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