Online Throughput Maximization in an Energy Harvesting Multiple Access Channel with Fading

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***Abstract*—In this paper, we consider an energy harvesting multiple access channel (MAC) where the transmitters are powered by energy harvested from the ambient environment. We assume that the energy harvesting processes at the transmitters can be modeled as independent Bernoulli processes with parameters λ_is, and the channel states between the transmitters and the receiver are independent Bernoulli processes with parameter μ_is. An active transmitter always transmits with a fixed power and consumes one unit amount of energy in a time slot. Under the assumption that $\mu_i \geq \lambda_i$, $\forall i$, our objective is to schedule the transmissions adaptively according to the instantaneous channel and battery states of transmitters, so that the long-term average sum-throughput of the MAC is maximized in expectation. We first show that for a general asymmetric scenario where λ_is and μ_is are not identical across the transmitters, the expected long-term average sum-throughput has an upper bound for any transmission scheduling policy satisfying the energy causality constraints. We then consider a special symmetric scenario where λ_is and μ_is are uniform among transmitters. We propose a randomized longest-connected-queue transmission scheduling policy and show that it achieves the upper bound almost surely as time T approaches infinity, thus it is optimal.**

***Index Terms*—energy harvesting, multiple access channel, scheduling**

I. Introduction

In order to build a self-sustainable wireless sensor network, powering sensor nodes with energy harvesting devices becomes a natural and feasible solution, thanks to the recent progress on energy harvesting technology. However, utilizing the random, scarce and non-uniform harvested energy adaptively to meet the energy demand from collecting and transmitting vast amounts of data in such networks is extremely challenging, and requires a completely different approach to energy management.

In this paper, we focus on the design of an *online* transmission scheduling policy for an energy harvesting multiple access channel (EH-MAC) with fading. Our objective is to maximize the long-term average sum-throughput through activating transmitters adaptively according to their channel states and battery levels in each time slot. We assume that energy arrives at individual transmitters according to independent Bernoulli processes with parameters λ_is, and an active transmitter always transmits with a fixed power and consumes one unit amount of energy in a time slot. Due to different channel states between the transmitters and the receiver, active transmitters may have different impacts on the sum-throughput in one transmission, which makes the problem very complicated. In order to make the problem tractable, as a first step, we assume that the channel states between the transmitters and the receiver are independent Bernoulli processes with parameters μ_is. The stochastic nature of the energy harvesting processes and the randomness of the channel states make the optimal transmission scheduling non-trivial. We first consider a general asymmetric setup where λ_is and $\mu_i s$ are not identical among the users. Under the assumption that $\mu_i \geq \lambda_i$, $\forall i$, the expected long-term average sum-throughput has an upper bound for any transmission scheduling policy satisfying the energy causality constraints. We then consider a symmetric scenario where λ_is and $\mu_i s$ are uniform among transmitters, and propose a randomized longest-connected-queue transmission scheduling policy. The policy achieves the upper bound almost surely as time T approaches infinity, thus it is optimal.

Throughput maximization in EH-MAC has been studied recently under various settings. In [1], a generalized backward waterfilling method is proposed to maximize the throughput region of a two-user EH-MAC under an offline setting. [2] investigates the offline sum-rate maximization problem for an N-user EH-MAC with fading and formulate it as a convex optimization problem, which is then solved by a low-complexity iterative dynamic water-filling algorithm. Offline throughput maximization with energy cooperation for EH-MAC has been studied in [4], [5]. Under an online setting, [3] studies a similar long-term average sum-throughput maximization problem in the continuous time regime with variable transmission power. The problem is formulated as partial integro-differential equations and solved by an iterative algorithm. [6]–[9] formulate the online throughput maximization problem for frequency-division EH-MAC as partially observable Markov decision processes (POMDP), under the assumption that the instantaneous states of the nodes' batteries are not available at the central controller.

Our setup is different from the previous work [1]–[9], as we consider a discrete-time online setting, and the instantaneous battery states of the transmitters are available at the central controller. The proposed queue-length based scheduling policy has a similar structure as the *longest-connected-queue* server

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allocation policies studied in [10], [11], however, it is also significantly different from them. First, we do not have restrictions on the number of active transmitters in each slot. The thresholding structure of our proposed policy is due to the properties of the sum-rate function of the multiple-access channel, rather than a hard constraint assumed in the system model. Second, the Lyapunov technique adopted in such work requires the incoming traffic rate vector to be strictly inside the network capacity in order to stabilize the system. However, the large deviation theory and sample path wise analysis adopted in this work do not have such requirement. The proposed randomized longest-connected-queue policy stabilizes the energy queues even if the energy harvesting rate λ_i is exactly equal to the probability that a transmitter is connected to the receiver in a slot, i.e., μ_i. Thus, our proposed policy provides a stronger system stability guarantee in this sense.

II. System Model and Problem Formulation

In this paper, we consider a multiple access channel consisting of N transmitters and a receiver, as shown in Fig. 1. Transmitters are powered by energy harvested from the ambient environment. Each transmitter is equipped with an infinite battery to store the harvested energy.

The channel from the transmitters to the receiver is additive white Gaussian noise (AWGN), i.e., $Y = \sum_{i=1}^{N} \sqrt{h_i} X_i + Z$, where Y is the received signal at the receiver, X_i is the transmit signal of transmitter i, h_i is the fading coefficient for the channel between transmitter i and the receiver, and Z is the Gaussian noise with zero-mean and unit-variance.

We consider a time-slotted system. In time slot t, a transmitter may transmit with a fixed power level P, or keep silent. We assume the energy is normalized so that exactly one unit amount of energy is spent in each slot for an active transmitter. We denote the subset of active transmitters in slot t as $\mathcal{C}_t$.

Considering a block-fading channel where the fading coefficients stay the same in each time slot, the sum-rate of the MAC in each slot t thus must satisfy

$$\sum_{i=1}^{N} R_i(t) \leq \frac{1}{2} \log\left(1 + \sum_{i:i\in\mathcal{C}_t} h_i(t)P\right) := f\left(\mathcal{C}_t, \mathbf{h}_t\right) \quad (1)$$

where $R_i(t)$ is the transmission rate of transmitter i in time slot t, $h_i(t)$ is the corresponding fading coefficient, and $\mathbf{h}_t := [h_1(t), h_2(t), \ldots, h_N(t)]$.

Let $E_i(t)$ denote the amount of energy remaining in the battery of node i at the beginning of time slot t, $A_i(t)$ be the amount of harvested energy at node i during slot t. For every sensor node i, we assume the energy arrival process is a Bernoulli process with parameter λ_i, $0 \leq \lambda_i \leq 1$, i.e., $\mathbb{E}[A_i(t)] = \lambda_i$. The arrival processes are independent and may not be identical across sensors. Assume the system starts with an empty state. Then, the energy queue evolves according to

$$\begin{aligned} &E_i(0) = 0, \forall i \\ &E_i(t+1) = E_i(t) - \mathbf{1}_{i\in\mathcal{C}_t} + A_i(t), \quad t = 0, 1, 2, \ldots, \forall i \end{aligned} \quad (2)$$

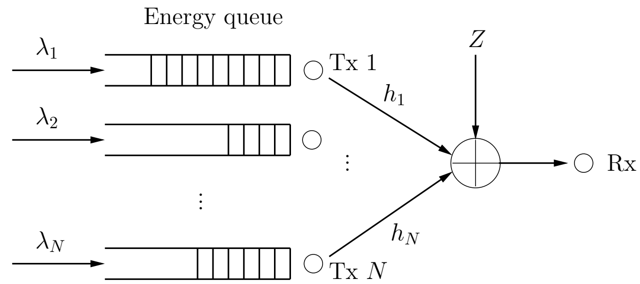

Fig. 1: An energy harvesting multiple access channel.

where $\mathbf{1}_x$ is an indicator function, i.e., it equals one if x is true, and it equals zero otherwise. Since an observation cannot be made if $E_i(t) < 1$, we impose the following energy constraint

$$E_i(t) \geq \mathbf{1}_{i\in\mathcal{C}_t}, \qquad \forall i, t. \quad (3)$$

Assuming the statistics of the energy harvesting processes are known at a central controller, our objective is to design an online transmission scheduling $\{\mathcal{C}_t\}_{t=1}^{\infty}$, such that the expected long-term average sum-throughput is maximized. The optimization problem can be formulated as

$$\max_{\{\mathcal{C}_t\}} \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t, \mathbf{h})\right] \quad \text{s.t. } (2)(3) \quad (4)$$

where the expectation in the objective function is taken with respect to all possible energy harvesting sample paths. The optimization problem in (4) is stochastic and has a combinatorial nature, and in general does not admit a closed-form solution.

In order to make the problem analytically tractable, as a first step, we assume that $\{h_i(t)\}_{t=1}^{\infty}$ is an independent Bernoulli process with parameter μ_i. When $h_i(t) = 1$, we say that transmitter i is connected to the receiver in slot t; otherwise, it is disconnected. In this case, even if $i \in \mathcal{C}_t$, it does not contribute to the sum-throughput. Based on such assumptions, we can simplify $f(\mathcal{C}_t, \mathbf{h})$ as $f(|\mathcal{C}_t^*|)$, where

$$\mathcal{C}_t^* := \{i | i \in \mathcal{C}_t, h_i(t) = 1\}. \quad (5)$$

In the following, we first analyze the optimization problem and provide an upper bound on the objective function for general λ_i and μ_i. We then consider a special symmetric scenario where $\lambda_i = \lambda$ and $\mu_i = \mu$, $\forall i$, and propose a randomized longest-connected-queue policy to achieve the upper bound.

III. An Upper Bound on the Objective Function

Define p_k as the probability that there exist exactly k connected transmitters in a time slot, i.e.,

$$p_k := \mathbb{P}\left[\sum_{i=1}^{N} h_i(t) = k\right] \quad (6)$$

and

$$g(k) := \mathbb{E}\left[\min\left(\sum_{i=1}^{N} h_i(t), k\right)\right] \quad (7)$$

Then, we have the following lemma.

Lemma 1 *Under the assumption that $\lambda_i \le \mu_i$, $\forall i$, there must exist an integer K and a q, $0 < q \le 1$, s.t.*

$$g(K)(1-q) + g(K+1)q = \sum_{i=1}^{N} \lambda_i \tag{8}$$

i.e.,

$$\sum_{i=1}^{K} p_i i + \left(\sum_{i=K+1}^{N} p_i\right)(K+q) = \sum_{i=1}^{N} \lambda_i \tag{9}$$

Proof: Based on the definition of $g(k)$ in eqn. (7), we have

$$g(k) = \sum_{i=1}^{k-1} p_i \cdot i + \sum_{i=k}^{N} p_i \cdot k \tag{10}$$

We note that $g(k)$ is monotonically increasing in k, with $g(0) = 0$ and $g(N) = \sum_{i=1}^{N} \mu_i \ge \sum_{i=1}^{N} \lambda_i$. Thus, there must exist a K such that

$$g(K) < \sum_{i=1}^{N} \lambda_i, \quad g(K+1) \ge \sum_{i=1}^{N} \lambda_i. \tag{11}$$

which implies (8) and (9). ■

Definition 1 *A transmission scheduling policy $\{\mathcal{C}_t\}_{t=1}^{\infty}$ is feasible if $E_i(t) \ge 1$, for every $i \in \mathcal{C}_t$, $\forall t$, i.e., the energy causality constraint (3) is always satisfied for every i, t.*

Lemma 2 *Under any feasible transmission scheduling policy,*

$$\limsup_{T\to+\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{1}_{i\in\mathcal{C}_t^*} \le \lambda_i \tag{12}$$

almost surely for all i, where $\mathcal{C}_t^*$ is defined in (5).

The proof of Lemma 2 is based on the energy queue evolution described in (2) and the strong law of large numbers, and it is omitted for the brevity of the presentation.

Lemma 3 *The objective function in (4) is upper bounded as*

$$\max_{\{\mathcal{C}_t\}} \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(|\mathcal{C}_t^*|)\right]$$
$$\le \sum_{k=1}^{K} p_k f(k) + \left(\sum_{k=K+1}^{N} p_k\right)\left[(1-q) f(K) + q f(K+1)\right],$$

where $\mathcal{C}_t^*$, p_k, q, and K are defined in (5), (6), and Lemma 1, respectively.

The proof of Lemma 3 relies on Lemma 1 and Lemma 2, and is omitted due to space limitation. The upper bound suggests a threshold on the active connected transmitters in each time slot, i.e., $|\mathcal{C}_t^*|$ should not exceed $K+1$ in any time slot. Besides, the portion of time that $|\mathcal{C}_t^*| = k, 1 \le k < K$ should equal to p_k, which by definition is the probability that k transmitters are connected to the receiver. This implies that if less than K transmitters are connected in a slot, then all of the connected transmitters should transmit, as if no energy constraints were imposed at them. Besides, the portion of time that $|\mathcal{C}_t^*| = K$ or $K+1$ should be carefully chosen, so that the long-term average of $|\mathcal{C}_t^*|$ is exactly $\sum_{i=1}^{N} \lambda_i$.

We note that in this upper bound, the energy causality constraints in (2)(3), even the long-term energy harvesting rate constraint on individual transmitters in (12), are not explicitly involved. The upper bound only depends on the sum of the energy harvesting rates $\sum_{i=1}^{N} \lambda_i$. This is equivalent to replace the original sample-path wise energy causality constraints with a more relaxed sum-power constraint instead. Thus, the question is: can the upper bound be achieved under the sample-path wise energy causality constraints for individual transmitters? and if so, how the transmissions should be coordinated, so that the EH-MAC asymptotically behaves like a MAC under equivalent sum-power constraint as $T \to \infty$?

The answers to those questions are not straightforward. In the next section, we study a special symmetric scenario, and show that the upper bound can indeed be achieved.

IV. Optimal Policy in a Symmetric Scenario

In this section, we consider a special symmetric scenario where $\lambda_i = \lambda$, and $\mu_i = \mu$, $\forall i$. For a MAC with identical energy harvesting and channel statistics for all transmitters, intuitively, balanced energy queue lengths are desirable. This is due to the fact that any queue can be equally likely to be among the $k, k < K$ connected queues in a future slot. In order to have all connected queues transmit in this case, they must have sufficient energy. Thus, balancing energy queue lengths as much as possible minimizes the battery outage probability of a connected queue. Motivated by this intuition, and the upper bound in Lemma 3, we propose a randomized longest-connected-queue policy as follows.

Definition 2 (Randomized Longest-Connected-Queue Policy) *In time slot t, if $\sum_{i=1}^{N} h_i(t) \le K$, all connected transmitters with sufficient energy will transmit; If $\sum_{i=1}^{N} h_i(t) > K$, sort all of the connected transmitters by their energy queue lengths in a descending order, and let ρ_t be an i.i.d random variable taking value $K+1$ with probability q and K with probability $1-q$. Then, the first ρ_t transmitters will transmit in time slot t if they have sufficient energy.*

Apparently, the randomized longest-connected-queue policy always provides a feasible transmission scheduling. With a little abuse of notation, we use ρ_t to denote be the *desired* number of active transmitters in each time slot t. Specifically, if $\sum_{i=1}^{N} h_i(t) \le K$, $\rho_t = \sum_{i=1}^{N} h_i(t)$; if $\sum_{i=1}^{N} h_i(t) > K$, $\rho_t = K+1$ with probability q, and $\rho_t = K$ with probability $1-q$.

Theorem 1 *In the symmetric scenario, the randomized longest-connected-queue policy achieves the upper bound on the long-term sum-throughput in Lemma 3 almost surely. Therefore, it is optimal.*

Corollary 1 *Under the randomized longest-connected-queue policy, for any sufficiently large T, we have*

$$\mathbb{P}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|\neq\rho_t}\geq\epsilon\right]\leq(T+1)^2\exp\left(-\frac{T\epsilon^2}{32N^4}\right) \quad (13)$$

The proof of Theorem 1 is provided in Appendix A. Theorem 1 indicates that the long-term average sum-throughput generated under the randomized longest-connected-queue policy converges to the upper bound, thus it is optimal. Corollary 1 implies that in almost every time slot, we have $|\mathcal{C}_t^*| = \rho_t$, i.e., as $T\to\infty$, the desired number of active transmitters can always be fulfilled. The corresponding convergence rate is explicitly characterized.

V. Numerical Results

In this section, we evaluate the performance of the proposed randomized longest-connected-queue transmission scheduling policy through simulations. We assume $P=1,\sigma^2=1$, thus $f(|\mathcal{C}_t^*|)=\frac{1}{2}\log(1+|\mathcal{C}_t^*|)$.

We first fix the number of transmitters in the system as $N=20$, the energy arrival rate $\lambda=0.3$, and the probability of a channel fading coefficient to be one as $\mu=0.5$ for every transmitter. Calculations indicate that $K=6$, $q=0.0298$. We then generate an energy harvesting sample path, and perform the proposed scheduling policy. We keep track of the total number of connected active transmitters (i.e., $|\mathcal{C}_t^*|$) under the policy, and plot its empirical distribution over the first 400 time slots in Fig. 2. We compare it with the theoretical distribution of ρ_t, i.e., the desired number of active transmitters under the policy. The discrepancy between them indicates that the desired number of active transmitter ρ_t has not been fulfilled in some time lots due to energy causality constraints at transmitters. However, the gap is not significant.

Then, we fix $\mu=0.7$, and vary the energy harvesting rate λ to be 0.3 and 0.5, respectively. We calculate the corresponding K and q for each setting, and evaluate the time average sum-throughput under the proposed randomized longest-connected-queue policy as a function of T. We generate 1,000 energy harvesting sample paths, and plot the sample average of $\frac{1}{T}\sum_{t=1}^{T}f(|\mathcal{C}_t^*|)$ in Fig. 3. We use the vertical bars to represent the 95% confidence intervals of the corresponding average sum-throughput. The results indicate that for a majority of the 1000 sample paths, the time average sum-throughput generated under the proposed policy converges to their corresponding upper limits quickly. When T is about 400, the gap between the upper bounds and the sample average of the sum-throughput becomes very small, which indicates the fast convergence rate of the proposed transmission scheduling policy.

Appendix

A. Proof of Theorem 1

Since $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}f(\rho_t)=\mathbb{E}[f(\rho_t)]$ almost surely, which equals the upper bound in Lemma 3, in order to prove Theorem 1, it suffices to prove that under the randomized longest-connected-queue policy,

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}f(|\mathcal{C}_t^*|)=\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}f(\rho_t), \quad a.s. \quad (14)$$

Thus, it suffices to prove that

$$\limsup_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|<\rho_t}=0, \quad a.s. \quad (15)$$

For a given T, define $\mathcal{A}=\left\{\sum_{i=1}^{N}E_i(T)>T\epsilon\right\}$. If $\forall\epsilon>0$, and T is sufficiently large, we have $\mathbb{P}[\mathcal{A}]$ decays exponentially in T, then by Borel-Cantelli lemma [12], we have $\mathbb{P}\left[\limsup_{T\to+\infty}\sum_{i=1}^{N}E_i(T)/T>\epsilon\right]=0$. This can then be used to prove (15).

At each time slot t, we reorder $E_i(t)$, $i=1,2,\dots,N$ according to their values, and denote $E_{(i)}(t)$ as the i-th largest one among them. For a given T, we define T_1 as the largest

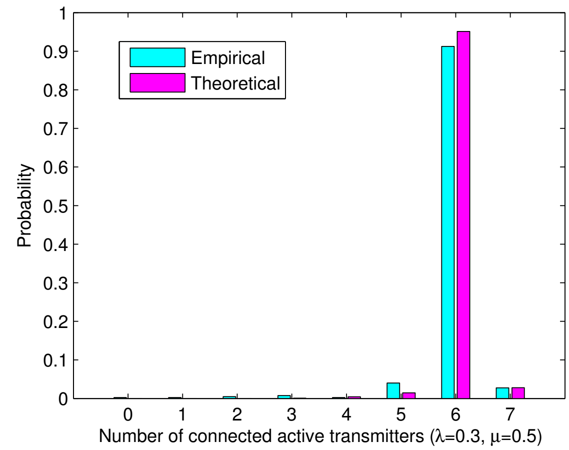

Fig. 2: The empirical distribution of $|\mathcal{C}_t^*|$ and the theoretical distribution of ρ_t.

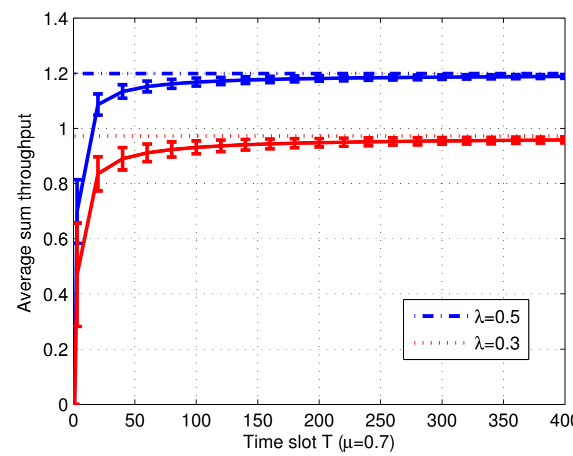

Fig. 3: The sample average of sum throughput $\frac{1}{T}\sum_{t=1}^{T}f(|\mathcal{C}_t^*|)$ as a function of time index T. Vertical bars represent 95% confidence intervals. Horizontal dotted lines indicate the corresponding upper bounds.

time index t, $t \leq T$, such that $E_{(N)}(t) = 0$, i.e., T_1 is the last time slot prior to T such that the shortest energy queue is zero. Then, $E_{(1)}(T_1)$ be the longest energy queue at T_1.

We partition event $\mathcal{A}$ into two sets $\mathcal{A}_1 := \{E_{(1)}(T_1) < \frac{T\epsilon}{2N}\} \cap \mathcal{A}$, and $\mathcal{A}_2 := \left\{E_{(1)}(T_1) \geq \frac{T\epsilon}{2N}\right\} \cap \mathcal{A}$, and bound their probabilities separately.

$$\mathbb{P}\left[\mathcal{A}_1\right] \leq \mathbb{P}\left[\sum_{i=1}^{N} E_i(T_1) \geq \frac{T\epsilon}{2}, \sum_{i=1}^{N} E_i(T) > T\epsilon\right] \tag{16}$$

$$\leq \mathbb{P}\left[\sum_{i=1}^{N} [E_i(T) - E_i(T_1)] > \frac{T\epsilon}{2}\right] \tag{17}$$

$$\leq \sum_{t_1=1}^{T-1} \mathbb{P}\left[\sum_{t=T_1+1}^{T} \left(\sum_{i=1}^{N} A_i(t) - \rho_t\right) > \frac{T\epsilon}{2}, T_1 = t_1\right] \tag{18}$$

$$\leq T \exp\left(-\frac{2(T\epsilon/2)^2}{TN^2}\right) = T \exp\left(-\frac{T\epsilon^2}{2N^2}\right) \tag{19}$$

where (16) follows from the definition of T_1, (17) follows from the triangle inequality, (18) follows from the fact that no energy queue is empty over $[T_1+1, T]$, and (19) follows from Hoeffding's inequality [13].

Next, we bound $\mathbb{P}[\mathcal{A}_2]$. Since for $\mathcal{A}_2$, we have $E_{(1)}(T_1) \geq \frac{T\epsilon}{2N}$ and $E_{(N)}(T_1) = 0$, there must exist an L such that

$$E_{(L)}(T_1) - E_{(L+1)}(T_1) \geq \frac{E_{(1)}(T_1)}{N-1} \geq \frac{T\epsilon}{2N(N-1)} \tag{20}$$

Assume the energy queue indices at time T_1 are (from the longest to the shortest) $l_1, \ldots, l_L, l_{L+1}, \ldots, l_N$. Let T_0 be the smallest time index t, $t \leq T_1$, s.t. $\forall t \in [T_0, T_1]$,

$$E_i(t) > E_j(t), \forall i \in \{l_1, \ldots, l_L\}, \forall j \in \{l_{L+1}, \ldots, l_N\} \tag{21}$$

Let i^* be the shortest queue in $\{l_1, l_2, \ldots, l_L\}$ at time T_0, and j^* be the longest queue in $\{l_{L+1}, \ldots, l_N\}$ at time T_0. Thus, we must have

$$E_{i^*}(t) > E_{j^*}(t) \quad \forall t \in [T_0, T_1] \tag{22}$$

Based on the definition of T_0, and the fact that an energy queue length can only increase or decrease by at most one in a single time slot, we have

$$E_{i^*}(T_0) - E_{j^*}(T_0) \leq 2 \tag{23}$$

On the other hand, (21) implies that

$$E_{i^*}(T_1) - E_{j^*}(T_1) \geq E_L(T_1) - E_{L+1}(T_1) \geq \frac{T\epsilon}{2N(N-1)} \tag{24}$$

Conditional on (22), we consider a random process $\{E_{i^*}(t) - E_{j^*}(t)\}_{t \in [T_0, T]}$. We can show that it is a super martingale, and the increment is always bounded by 2.

Thus,

$$\mathbb{P}[\mathcal{A}_2] \leq \mathbb{P}[(22), (23), (24)] \tag{25}$$

$$\leq \mathbb{P}\left[\Delta \geq \frac{T\epsilon}{2N(N-1)} - 2, (22)\right] \tag{26}$$

$$\leq \sum_{t_1=1}^{T} \sum_{t_0=1}^{t_1} \mathbb{P}\left[\Delta \geq \frac{T\epsilon}{2N^2}, T_1 = t_1, T_0 = t_0 \middle| (22)\right] \tag{27}$$

$$\leq T(T-1) \exp\left(-\frac{T\epsilon^2}{32N^4}\right) \tag{28}$$

where $\Delta := [E_{i^*}(T_1) - E_{j^*}(T_1)] - [E_{i^*}(T_0) - E_{j^*}(T_0)]$, and (28) follows from Azuma-Hoeffding's inequality.

Combining (19) and (28), we have

$$\mathbb{P}[\mathcal{A}] = \mathbb{P}[\mathcal{A}_1] + \mathbb{P}[\mathcal{A}_2] \leq T^2 \exp\left(-\frac{T\epsilon^2}{32N^4}\right) \tag{29}$$

when T is sufficiently large. Therefore, $\sum_{i=1}^{N} E_i(T)/T \to 0$ almost surely. This implies that $\lim_{T\to+\infty} \frac{1}{T} \sum_{t=1}^{T} |\mathcal{C}_t^*| = \lim_{T\to+\infty} \frac{1}{T} \sum_{t=0}^{T} \sum_{i=1}^{N} A_i(t)$. On the other hand, the strong law of large numbers indicates that $\lim_{T\to+\infty} \frac{1}{T} \sum_{t=1}^{T} \rho_t = \sum_{i=1}^{N} \lambda_i$ almost surely. Thus, $\lim_{T\to+\infty} \frac{1}{T} \sum_{t=1}^{T} |\mathcal{C}_t^*| = \lim_{T\to+\infty} \frac{1}{T} \sum_{t=1}^{T} \rho_t$ almost surely, which implies (15) and completes the proof.

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