Optimal Online Sensing Scheduling for Energy Harvesting Sensors With Infinite and Finite Batteries

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***Abstract*—In this paper, we study the optimal sensing scheduling problem for an energy harvesting sensor. The objective is to strategically select the sensing time such that the long-term time-average sensing performance is optimized. In the sensing system, it is assumed that the sensing performance depends on the time durations between two consecutive sensing epochs. Example applications include reconstructing a wide-sense stationary random process by using discrete-time samples collected by a sensor. We consider both scenarios where the battery size is infinite and finite, assuming the energy harvesting process is a Poisson random process. We first study the infinite battery case and identify a performance limit on the long-term time average sensing performance of the system. Motivated by the structure of the performance limit, we propose a best-effort uniform sensing policy, and prove that it achieves the limit asymptotically, thus it is optimal. We then study the finite battery case, and propose an energy-aware adaptive sensing scheduling policy. The policy dynamically chooses the next sensing epoch based on the battery level at the current sensing epoch. We show that as the battery size increases, the sensing performance under the adaptive sensing policy asymptotically converges to the limit achievable by the system with infinite battery, thus it is asymptotically optimal. The convergence rate is also analytically characterized.**

***Index Terms*—Energy harvesting, finite battery, best-effort uniform sensing scheduling, adaptive sensing scheduling.**

I. Introduction

In THIS paper, we investigate the optimal online sensing scheduling of an energy harvesting sensor. Energy arrives at the sensor according to a Poisson process, and a unit amount of energy is consumed by the sensor to collect one measurement. A sensor cannot take any measurement if it does not have sufficient energy in its battery, i.e., sensing operations must satisfy the energy causality constraint. We consider an application scenario where a sensor collects measurements at discrete time epochs to estimate a time evolving physical quantity (temperature, humidity, etc). Modeling the monitored quantity as a random process, we assume that the sensing performance is a function of the discrete sensing epochs. Then, the question we aim to answer is: *Given the statistics of the energy harvesting process, how would the system strategically select the sensing epochs to optimize the long-term expected sensing performance, subject to the energy causality constraint at the sensor?* Ideally, the sensing policy should be online, lightweight, and require minimum knowledge of the energy harvesting process and the underlying monitored random process.

There are three dimensions of difficulty in designing such a sensing policy. First, the scarce energy supply imposes a stringent constraint on the number of measurements the sensor can take. In order to make each sample count, the sensing policy need to exploit the structural properties of the underlying monitored random process. Second, the energy harvesting process is stochastic in nature. The sensing policy should be able to cope with the fluctuations in energy supply and maintain a reliable sensing performance for almost all possible energy harvesting profiles. Third, in most practical scenario, a sensor is equipped with a finite battery, and energy overflow may happen if it is not spent in time. The sensor thus faces a dilemma of spending energy to collect less informative samples, or of saving energy for more advantageous time epochs, a step which may lead to energy loss.

In this paper, we consider a special sensing performance function, which is motivated by the form of time-average error in reconstructing a random process with power-law decaying covariance [1]. We exploit the properties of the sensing performance function to devise our online sensing scheduling policy. We investigate both cases when the battery size is infinite and finite. When the battery size is infinite, we first identify a performance limit on the long-term time average sensing performance of the system. Motivated by the structure of the performance limit, we propose a best-effort uniform sensing policy, and prove that it achieves the limit asymptotically, thus it is optimal. When the battery size is finite, we aim to investigate the impact of finite battery size on the sensing performance, and bring the sensing performance as close to that of the system with infinite battery as possible. We propose an energy-aware adaptive sensing scheduling policy, which dynamically chooses the next sensing epoch based on the battery level at the current sensing epoch, and show that it is asymptotically optimal as the battery size increases. The convergence rate is also explicitly characterized.

A. Main Contribution

The main contributions of this paper are threefold:

1) First, we study an application oriented sensing scheduling for energy harvesting sensors. Different from most existing energy management schemes where the optimization

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objective function depends on the instantaneous power allocated to the sensor, in our formulation, the sensing performance depends on the durations between consecutive sensing epochs. Thus, instead of deciding the instantaneous power consumption over the whole operation duration, in this paper, the objective is to decide the discrete sensing epochs for the sensor under the energy constraints. Such formulation is fundamentally different from existing works. It requires a new set of analytical tools, and results in a different type of energy management policies.

2) Second, we investigate both the infinite battery case and the finite battery case, and propose two intuitive yet practical online sensing scheduling policies with provable performance guarantees. The proposed scheduling policies only require the instantaneous battery level to decide the sensing epochs. Thus, the sensor can be turned off between two scheduled sensing epochs to save energy. This is extremely helpful for sensors operating under stringent energy constraint. For the finite battery case, we explicitly identify the convergence rate of the proposed policy as a function of the battery size, which provides theoretical guidelines on the system design of energy harvesting sensing systems.
3) Finally, we introduce Martingale process, renewal process, and a novel virtual energy harvesting sensing system to analyze the battery level evolution under the proposed policies. Such mathematical tools are new to the area of energy harvesting communications and networks, and might be useful for related problems, especially for the construction and analysis of online scheduling policies.

B. Related Work

A large number of energy management schemes have been proposed to cope with the random nature of the energy supply at energy harvesting sensors from different perspectives. Under the infinite battery assumption, energy management schemes have been developed to optimize communication related metrics, such as channel capacity, transmission delay or network throughput [3]–[5], and signal processing related performance metrics, such as estimation mean squared error (MSE), detection delay, false alarm probability [6], [7].

When the finite battery assumption is imposed, it changes the problem dramatically, and makes the corresponding optimal energy management much more complicated. One approach is to formulate the energy management problem as a one-shot offline optimization problem, under the assumption that the energy harvesting profile is known in advance. Examples include the throughput maximization problems studied in [8]–[10], where the optimal policies are significantly different from their counterparts in an infinite battery setting [3], [4], [11]. Another approach is to formulate the optimal energy management problem as an online stochastic control problem, assuming that only the statistics and the history of the energy harvesting process are available at the controller. Modeling the energy replenishing process as a Markov process, [12] aims to maximize the time average reward by making decisions regarding whether to transmit or discard a packet based on the current energy level. The optimal policy is shown to have a threshold structure. [13] studies the performance limits of a sensing system where the battery size and the data buffer are finite and proposes an asymptotically optimal energy management scheme. The dynamic activation of sensors with unit battery in order to maximize the sensing utility is studied in [14]. In general, online optimal energy management policies under a finite battery constraint are often very difficult to characterize. Explicit solutions only exist for certain special scenarios.

The finite battery case studied in this paper is significantly different from that in [13]. [13] considers a time-slotted system, and the objective is to adaptively vary the amount of energy spent in each time slot to optimize the system performance. However, we consider a continuous-time system in this paper, and the proposed asymptotically optimal design varies the durations between two consecutive sensing epochs according to the instantaneous battery level. This makes the analysis of the system performance under the proposed policy much more challenging.

C. Paper Outline

This paper is organized as follows. Section II states the system model and problem formulation. Section III provides the sensing scheduling policy for the infinite battery case and proves its optimality. Section IV describes an adaptive sensing scheduling policy for the finite battery case, and analytically characterizes its performance. Simulation results are provided in Section V. Section VI concludes the paper. Proofs of the main theorems are presented in the Appendix.

II. System Model and Problem Formulation

A. Energy Harvesting Model

Consider a sensor node powered by energy harvested from the ambient environment. It is assumed that the sensor node has an energy queue, such as a rechargeable battery or a super capacitor, to store the harvested energy. The energy queue is replenished randomly and consumed by taking observations. It is assumed that a unit amount of energy is required for one sensing operation. Without loss of generality, we assume the sensor is equipped with a battery with capacity B, $B \geq 1$. When $B = \infty$, it corresponds to the infinite battery case.

The energy arrival follows a Poisson process with parameter λ. Hence, energy arrivals occur in discrete time instants. Specifically, we use $t_1, t_2, \ldots, t_n, \ldots$ to represent the energy arrival epochs. Then, the energy inter-arrival times $t_i - t_{i-1}$ are exponentially distributed with mean λ. We assume $\lambda = 1$ throughout this paper for ease of exposition. If $\lambda \neq 1$, we can always normalize the time axis to make the energy arrival rate equal to one unit per unit time, and the algorithms and theoretical results presented in this paper will still be valid on the normalized time scale. Without loss of generality, it is assumed that the system starts with an empty energy queue at time 0.

A sampling policy or sensing scheduling policy is denoted as $\{l_n\}_{n=1}^{\infty}$, where l_n is the n-th sensing time instant. Let $l_0 =$

0, and $d_n := l_n - l_{n-1}$, for $n = 1, 2, \ldots$. Define $A(d_n)$ as the total amount of energy harvested in $[l_{n-1}, l_n)$, and $E(l_n^-)$ as the energy level of the sensor right before the scheduled sensing epoch l_n. Then, under any feasible sensing scheduling policy, the energy queue evolves as follows

$$E(l_{n+1}^-) = \min\{E(l_n^-) - 1 + A(d_{n+1}), B\} \tag{1}$$

$$E(l_n^-) \geq 1 \tag{2}$$

for $n = 1, 2, \ldots$. Eqn. (2) corresponds to the energy causality constraint in the system. Based on the Poisson arrival process assumption, $A(d_{n+1})$ is an independent Poisson random variable with parameters d_{n+1}.

B. Sensing Performance Metric

We assume the sensing performance depends on how the sensing epochs are placed in time. Given that the durations between two sensing epochs are d_n, $n = 1, 2, \ldots$, the sensing performance over the sensing period is measured by $\sum_n f(d_n)$. In addition, we make the following assumptions.

Assumptions 1 The sensing performance function $f(d)$, $d > 0$, has the following properties:

1) $f(d)$ is convex and monotonically increasing in d.
2) $f(d)/d$ is increasing in d.
3) $f(d) \leq Cd$, where C is a positive constant.

One example application that fits this model is to use samples collected at discrete time instants to estimate a time evolving physical quantity (temperature, humidity, etc), which is modeled as a random processes with power-law decaying covariance. It is shown that the linear minimum MSE (MMSE) estimation for any point on the random process only requires the two adjacent discrete-time samples bounding the point [1]. In this case, $f(d)$ can be interpreted as the total MSE over a length-d interval bounded by two consecutive sensing epochs. Optimizing the overall sensing performance is equivalent to minimizing the total MSE of the linear MMSE over the whole sensing period.

Such assumptions enable us to bound the long-term average sensing performance and motivate the design of the optimal sensing policies. We point out that in this paper, we require d to be strictly greater than zero, i.e., we do not consider the scenario where multiple samples are collected at the same time point. This is because if multiple samples are collected at a time, in general, the long-term sensing performance will depend on the number of samples collected at individual sensing epochs, as well as the durations between them. Therefore, it may not reasonable to assume that the sensing performance over the sensing period can be decomposed into the form of $\sum_n f(d_n)$. We will examine specific forms of sensing performance functions to accommodate such sensing operations, and explore the optimal sampling policy in this scenario in the future.

For a clear exposition of the result, we assume that two samples at time 0 and time T are available at the sensor for free, i.e., no energy is used for collecting those two samples. Denote these two sampling epochs as $l_0 = 0, l_{N_T+1} = T$. Besides, there are N_T sensing epochs placed over $(0, T)$. The overall sensing performance over the duration $[0, T]$ is then a summation of $f(d_n)$, $n = 1, 2, \ldots, N_T + 1$.

C. Problem Formulation

Our objective is to optimize the long-term average sensing performance by strategically selecting the sensing epochs $\{l_n\}_{n=1}^{\infty}$. We restrict to online policies, i.e., whenever the system decides a sensing epoch, its decision only depends on the energy harvesting profile up to that time, as well as previous sensing decisions. The optimization problem is formulated as

$$\begin{aligned} \min_{\{l_n\}_{n=1}^{\infty}} . \quad & \limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\ \text{s.t.} \quad & (1)-(2) \end{aligned} \tag{3}$$

where the expectation in the objective function is taken over all possible energy harvesting sample paths.

This is essentially a stochastic control problem. In contrast to other discrete-time stochastic control problems where decisions need to be made at every time slot (e.g., Markov Decision Process (MDP)), in this paper, we consider a continuous time setting, and decisions can be made at arbitrary time points. Actually, as we will see in Sec. IV, selecting the decision points could be a task for the scheduler as well. Therefore, this problem does not admit an MDP formulation in general, and it is extremely challenging to explicitly identify the optimal solution.

III. Sensing Scheduling With Infinite Battery

In this section, we will study the optimal sensing scheduling for the infinite battery case. We will show that the sensing performance (i.e., time-average MSE) in this scenario has a lower bound, which can be achieved almost surely by a best-effort uniform sensing scheduling policy. The performance limit provided in this section, and the best-effort uniform sensing algorithm will serve as a baseline for the finite battery case discussed in Section IV.

Lemma 1: Under every feasible scheduling policy, we have

$$\limsup_{T\to+\infty} \frac{N_T}{T} \leq 1, \quad a.s. \quad \forall i \tag{4}$$

where $N_T = \sum_{n=1}^{\infty} \mathbf{1}_{l_n \leq T}$ is the total number of samples taken in $[0, T]$.

Proof: Due to the energy causality constraint (2), we always have $N_T \leq \sum_{n=1}^{\infty} \mathbf{1}_{t_n \leq T}$, therefore

$$\limsup_{T\to+\infty} \frac{N_T}{T} \leq \limsup_{T\to+\infty} \frac{\sum_{n=1}^{\infty} \mathbf{1}_{t_n \leq T}}{T} = 1 \quad a.s.$$

where the last equality follows from the strong law of large numbers. ■

Lemma 2: The objective function in (3) is lower bounded as

$$\limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \geq f(1) \tag{5}$$

Proof:

$$\begin{aligned}
&\limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\
&\quad \geq \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\
&\quad \geq \mathbb{E}\left[\liminf_{T\to+\infty}\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] && (6)\\
&\quad \geq \mathbb{E}\left[\liminf_{T\to+\infty}\frac{N_T+1}{T} f\left(\frac{\sum_{n=1}^{N_T+1} d_n}{N_T+1}\right)\right] && (7)\\
&\quad = \mathbb{E}\left[\liminf_{T\to+\infty}\frac{N_T+1}{T} f\left(\frac{T}{N_T+1}\right)\right] \geq f(1) && (8)
\end{aligned}$$

where (6) follows from Fatou's Lemma, (7) follows from the convexity of f. The last inequality in (8) follows from Lemma 1 and the assumption that $f(d)/d$ is an increasing function in d. ■

Definition 1 (Best-effort Uniform Sensing Scheduling) The sensor is scheduled to perform the sensing task at $s_n = n$, $n = 1, 2, \ldots$. The sensor performs the sensing task at s_n if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled sensing epoch.

Here we use s_n to denote the n-th *scheduled* sensing epoch, which is in general different from the n-th *actual* sensing epoch l_n since some of the scheduled sensing epochs may be infeasible.

Theorem 1 Under the best-effort uniform sensing scheduling policy, we have

$$\lim_{T\to+\infty}\frac{N_T}{T} = 1 \quad a.s.$$

The proof of Theorem 1 is provided in Appendix A. Theorem 1 indicates that the best-effort uniform sensing scheduling policy is asymptotically equivalent to a uniform sensing policy almost surely, i.e., the sensor has sufficient energy to perform the task for almost every scheduled sensing epoch.

Theorem 2 The best-effort uniform sensing scheduling policy is optimal when the battery size is infinite, i.e.,

$$\limsup_{T\to+\infty}\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n) = f(1) \quad a.s.$$

where d_n is the duration between the actual sensing epochs l_n and l_{n-1}.

The proof of Theorem 2 is provided in Appendix B. Theorem 2 indicates that for almost every energy harvesting sample path, the best-effort uniform sensing policy converges to the lower bound in Lemma 2 when the battery size is infinite. This is due to the fact that when the battery size is infinite, the fluctuations of the energy arrivals can be averaged out when time is sufficiently large, thus a uniform sensing scheme with sensing rate equal to the energy harvesting rate can be achieved asymptotically as T is sufficiently large. Thus, the proposed best-effort uniform sensing is optimal. However, with finite battery, it may not be able to achieve the lower bound, since energy overflow is inevitable in this situation, which in turn results in more frequent infeasible sensing epochs due to battery outage.

IV. Sensing Scheduling With Finite Battery

In order to optimize the sensing performance when the battery size is finite, intuitively, the sensing policy should try to prevent any battery overflow, as wasted energy leads to performance degradation. Meanwhile, the properties of the sensing performance function require the sensing epochs to be as uniform as possible. Those two objectives are not aligned with each other, thus, the optimal scheduling policy should strike a balance between them.

In the following, we propose an energy-aware adaptive sensing scheme. Different from the best-effort uniform sensing scheduling policy that schedules the sensing epochs uniformly, the proposed sensing policy adaptively changes its sensing rate based on the instantaneous battery level. Intuitively, when the battery level is high, the sensor should sense more frequently in order to prevent battery overflow; When the battery level is low, the sensor should sense less frequently to avoid infeasible sensing epochs. Meanwhile, the sensing rate should not vary significantly so that a relatively uniform sensing scheduling can be achieved.

Definition 2 (Energy-aware Adaptive Sensing Scheduling) The adaptive sensing scheduling policy defines sensing epochs s_n recursively as follows

$$s_n = s_{n-1} + \begin{cases} \frac{1}{1-\beta}, & E(s_{n-1}^-) < \frac{B}{2} \\ 1, & E(s_{n-1}^-) = \frac{B}{2} \\ \frac{1}{1+\beta}, & E(s_{n-1}^-) > \frac{B}{2} \end{cases} \tag{9}$$

where $s_0 = 0$, $E(s_0^-) = 1$, and

$$\beta := \frac{k\log B}{B} \tag{10}$$

with k being a positive number such that $0 < \beta < 1$. The sensor performs the sensing task at s_n if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled sensing epoch.

Remark 1: The policy divides the battery state space into three different regimes. At each scheduled sensing epoch, the sensor decides whether to sense according to its current battery state, and adaptively selects the next sensing epoch depending on which regime the current battery state falls in. When it is above $B/2$, the sensor senses every $\frac{1}{1+\beta}$ units of time, and when it is below $B/2$, it senses every $\frac{1}{1-\beta}$ units of time. The value of β controls the deviation of the sensing rates. Intuitively, when the value of β increases, the probability that the battery overflows decreases, so does the probability that a scheduled sensing epoch is infeasible. However, larger β may also lead to large variation in the durations between consecutive sensing epochs, which results in sensing performance degradation.

Remark 2: We note that the scheduled sensing epochs are defined in a recursive fashion. At each scheduled sensing epoch, the sensor only need to check its current battery level and decide the next sensing epoch. Thus, the sensor can be turned off temporarily until the next sensing epoch. This could save a significant amount of energy of the sensor from staying awake and constantly monitoring the battery status.

Remark 3: As $B \to \infty$, we have $\beta \to 0$ for any fixed k, i.e., the adaptive sensing policy converges to the best-effort uniform sensing proposed in Section III as the battery size increases. Thus, it is reasonable to expect that the adaptive sensing policy is asymptotically optimal as the battery size approaches infinity.

In the following two theorems, we prove the asymptotical optimality of the adaptive sensing policy, and characterize the speed of its convergence analytically.

Theorem 3 Over the sensing period $(0, T)$, we denote $A(T)$ as the total amount of harvested energy, N'_T as the total number of scheduled sensing epochs, and N_T as the total number of actual sensing epochs as defined previously in Section II. Then, under the adaptive sensing scheduling policy, the ratio of infeasible sensing epochs, denoted as $\lim_{T\to\infty} \frac{N'_T - N_T}{N'_T}$, scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$, and the average amount of wasted energy per unit time, denoted as $\lim_{T\to\infty} \frac{A(T)-N_T-E(T)}{T}$ scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$.

Theorem 3 indicates that when B is sufficiently large, both upper bounds of the battery outage and overflow probabilities decrease monotonically as k increase. As the battery size B increases, the upper bounds of those two probabilities decrease and eventually approach zero. Thus, the proposed policy is asymptotically equivalent to a uniform sensing policy, similar to the best-effort uniform sensing policy for the infinite battery case.

Theorem 4 Under the adaptive sensing scheduling policy, the gap between the time average sensing performance, denoted as $\lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{N_T+1} f(d_n)$, and the lower bound $f(1)$ scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$.

Theorem 4 implies that as battery size B increases, the sensing performance under the adaptive sensing scheduling policy approaches the lower bound achievable for the system with infinite battery. Thus, it is asymptotically optimal. Compared to the bounds in Theorem 3, the bound in Theorem 4 has an extra term $\left(\frac{\log B}{B}\right)^2$. For a sufficiently large B, the bound is dominated by the first term when k is small, and it is dominated by the second term when k is large. Thus, it may not monotonically decrease as k increase, which is consistent with the fact that the sensing performance is not only related to the battery outage and overflow probabilities, but also depends on the durations between consecutive sensing epochs.

The proofs of Theorems 3 and 4 are provided in Appendices D and E, respectively. The sketch of the proof is as follows. The battery states at scheduled sensing epochs form a discrete-time random process $\{E(s_n^-)\}_{n=1}^{\infty}$. However, it differs from a conventional discrete-time random process since the duration between two consecutive time indices varies in time: it could be $\frac{1}{1-\beta}$, $\frac{1}{1-\beta}$ or 1, depending on the battery state. This makes the analysis very complicated. To simplify the analysis, in Appendix C, we construct a "virtual" energy harvesting sensing system, whose battery state can be any integer in $(-\infty, +\infty)$. Assuming the virtual sensing system senses at a uniform rate, we analytically characterize the expected duration between two consecutive events that the virtual battery state hits a certain level. We then consider the portion of $\{E(s_n^-)\}_{n=1}^{\infty}$ lying in $(0, B/2]$ and $[B/2, B)$ separately. In Appendix D, we show that the portion lying in each region can be mapped to a virtual system, and exploit the analytical results in Appendix C to prove Theorem 3. In Appendix E, we use the results from Appendix D and the properties of the sensing performance function $f(d)$ to prove Theorem 4.

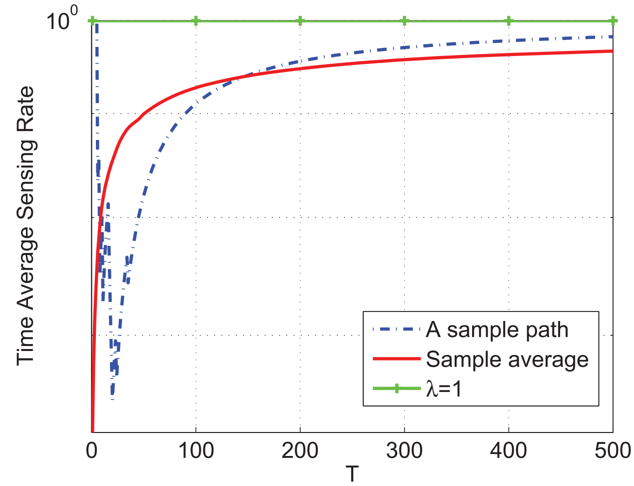

Fig. 1. Sensing rate as a function of T.

V. Simulation Results

The performances of the proposed sensing scheduling policies are evaluated in this section through simulations. We adopt the MSE function for random process reconstruction in [1] to measure the sensing performance under the proposed sensing scheme. Specifically, the correlation between two samples separated by a time duration d is ρ^d, and the average reconstruction MSE of the random field between two d-spaced samples is

$$f(d) = d\frac{1+\rho^{2d}}{1-\rho^{2d}} + \frac{1}{\log \rho} \tag{11}$$

The power-law parameter ρ is set to be 0.7 in the simulations.

First, we evaluate the uniform best-effort sensing policy for the infinite battery case. We generate 1,000 energy harvesting profiles according to the Poisson random process with $\lambda = 1$, and perform the best-effort uniform sensing for each energy harvesting profile. The sensing rate, N_T/T, for each energy harvesting profile is tracked and recorded. One sample path and the sample average sensing rate for the 1,000 sample paths are plotted as functions of T in Fig. 1. It is observed that the sensing rate approaches $\lambda = 1$ asymptotically as T increases, as predicted in Theorem 1. Thus the best-effort sampling policy can almost surely approach the behavior of uniform sampling when $T > 400$.

The sensing performance under the best-effort uniform sensing policy is shown in Fig. 2. Again, we plot one sample path and the sample average over the 1,000 sample paths of the time average sensing performance as functions of T in the figure. We observe that the sensing performance curves gradually approach the lower bound $f(1)$ as T increases. When

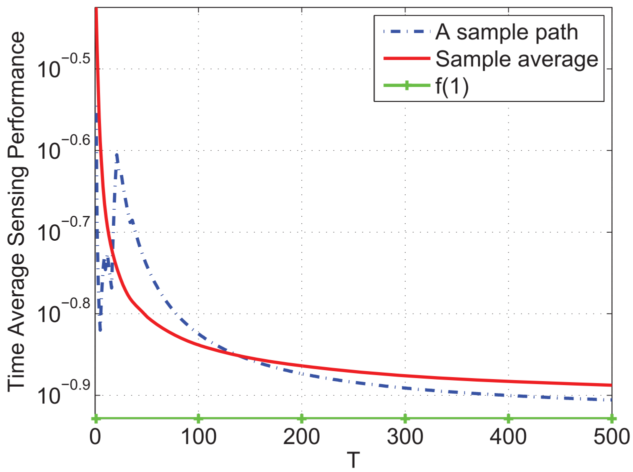

Fig. 2. Sensing performance as a function of T.

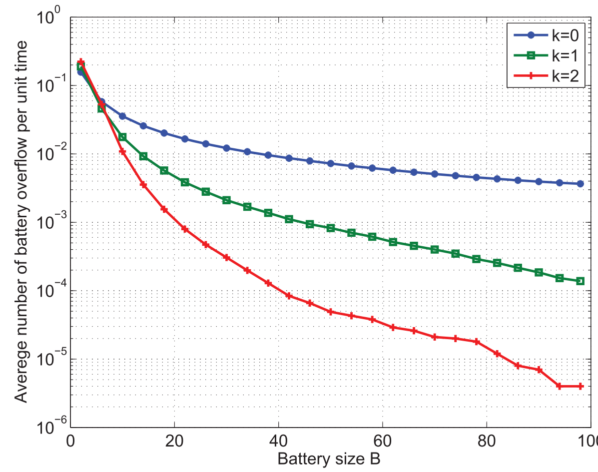

Fig. 4. The average number of battery overflow per unit time.

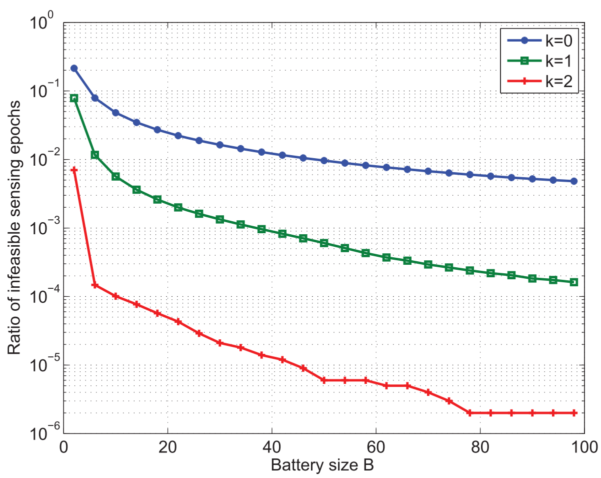

Fig. 3. The ratio of infeasible sensing epochs.

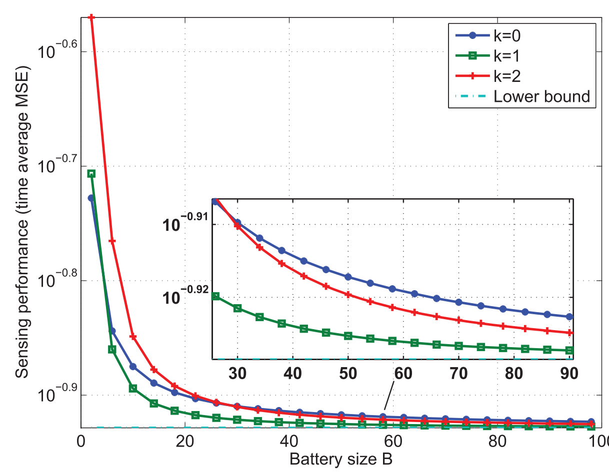

Fig. 5. The time averaged sensing MSE.

$T = 500$, there is only a very small difference between the simulation results and the analytical lower bound. The results indicate that the proposed best-effort uniform sensing policy is asymptotically optimal.

Next, we evaluate the adaptive sensing scheduling policy for the finite battery case. Fixing the energy harvesting rate to be $\lambda = 1$ per unit time, and $T = 100,000$, we generate a sample path for the Poisson energy harvesting process, and perform the sensing according to the policy. We keep track of the following quantities. First, we count the total number of scheduled sensing epochs under the policy. Among those scheduled sensing epochs, we count the total number of infeasible ones (i.e., the epoch s_n when $E(s_n^-) < 1$), record the ratio of infeasible sensing epochs under the policy. We let $k = 0, 1, 2$, respectively, and perform the adaptive sensing according to (9) with battery size B varying from 2 to 100. The corresponding results are plotted in Fig. 3. We note that for each fixed k, the ratio monotonically decreases as B increase, and each curve is roughly convex in B. This is consistent with the theoretical bounds in Theorem 3. Meanwhile, for each fixed battery size, the ratio decreases as k increases. This is due to the fact that the adaptive sensing policy is more conservative for larger k when battery level is below $B/2$, i.e., it senses at a slower rate for larger k, which makes the energy level drift away from empty state with higher probability.

Next, we study battery overflow under the proposed policy. We count the total number of time instants when the battery state exceeds B, and divide it by T. The average number of battery overflow events per unit time is plotted as a function of B in Fig. 4 for $k = 0, 1, 2$, respectively. Again, we observe that for each fixed k, the curve is monotonically decreasing and roughly convex in B, as predicted by the theoretical bounds in Theorem 3. Meanwhile, for each fixed battery size, the battery overflow rate decreases as k increases. This is due to the fact that the adaptive sensing policy is more aggressive for larger k when battery level is above $B/2$, i.e., it senses at a faster rate for larger k. Thus, the energy level drifts away from full state with higher probability.

At last, we study the sensing performance in terms of the time averaged MSE. We calculate the MSE for each interval bounded by two consecutive sensing epochs as (11), aggregate them and divide the sum by T. The time averaged MSE is plotted in Fig. 5. We note that for each fixed k, the gap between the time averaged MSE and the lower bound monotonically decreases as B increases, which is consistent with the theoretical result in Theorem 4. However, when B is fixed, the best sensing performance is observed at $k = 1$, which is different from the results in Figs. 3 and 4. Even though the battery outage and overflow rates decrease in k, the average sensing performance does not exhibit such monotonicity. This is because when k is large, the sensing rate varies dramatically in time. Although this leads to lower outage and overflow probabilities, it compromises the sensing performance as the sensing scheduling deviates from the desired uniform sensing scheduling. Thus, there exists a tradeoff between reducing battery outage and overflow probabilities, and equalizing the sensing rates. The optimal selection of k should jointly consider those two conflicting objectives.

VI. Conclusions

In this paper, we considered the optimal online sensing scheduling policy for an energy harvesting sensing system. We first provided a lower bound on the time averaged sensing performance for the system with infinite battery, and showed that this lower bound can be achieved by a best-effort uniform sensing policy. We then investigated the finite battery case and proposed an energy-aware adaptive sensing scheduling policy, which dynamically varies the sensing rate based on instantaneous energy level of the battery. We showed that the battery outage and overflow probabilities under the proposed policy approach zero as the battery size goes to infinity, and the time averaged sensing performance converges to the lower bound when the battery size increases. Thus the adaptive sensing scheduling policy is asymptotically optimal. The convergence rate as a function of the battery size was also explicitly characterized. Simulation results validated the theoretical bounds.

Appendix

A. Proof of Theorem 1

The best-effort uniform sensing policy partition the time axis into slots, each with length 1. Consider the number of energy arrivals during a slot, denoted as A. Due to the Poisson process assumption of the energy arrival process, we have

$$\mathbb{P}[A=k]=\frac{e^{-1}}{k!},\quad k=0,1,2\ldots$$

Let $E(n)$ be the energy level of the sensor right before the scheduled sensing epoch n. Based on $E(n)$, we can group the time slots into segments with lengths $u_0, v_1, u_1, \ldots, v_k, u_k, \ldots$, where u_is correspond to the segments that begin with $E(n)=0$ and v_is correspond to the segments that begin with $E(n)>0$, as shown in Fig. 6. $E(n)$ jumps from zero to some positive value e_i at the end of the segment corresponding to u_i. Therefore, u_i follows an independent geometric distribution

$$\mathbb{P}[u_i=k]=e^{-(k-1)}\left(1-e^{-1}\right),\quad k=1,2\ldots$$

and v_i follows a "random walk" with increment $A-1$ starting at some positive level e_i until it hits 0. Note that v_i contains a random walk Γ_i which starts at e_i and finishes at e_i-1 for the first time. Denote the duration of Γ_i as τ_i.

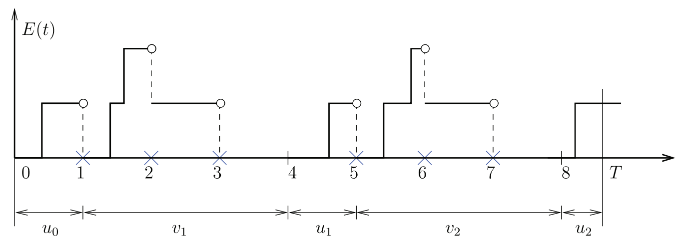

Fig. 6. An energy level evolution sample path. Crosses represent actual sensing epochs.

Let K_T be the number of segments with $E(n)=0$ during T. Note that $T=N_T+\sum_{i=0}^{K_T}u_i$. Therefore, to show $N_T/T\to 1$ almost surely, it suffices to show that

$$\lim_{T\to\infty}\frac{\sum_{i=0}^{K_T}u_i}{T}=0,\quad a.s.$$

Note that

$$\frac{\sum_{i=0}^{K_T}u_i}{T}=\frac{\sum_{i=0}^{K_T}u_i}{K_T}\frac{K_T}{T}\leq\frac{\sum_{i=0}^{K_T}u_i}{K_T}\frac{K_T}{\sum_{i=1}^{K_T}\tau_i}$$

As we will show in the following, $K_T\to\infty$ almost surely as $T\to\infty$. Then, by the strong law of large numbers,

$$\lim_{T\to\infty}\frac{\sum_{i=0}^{K_T}u_i}{K_T}=\frac{1}{1-e^{-1}},\quad a.s.$$

Therefore, to prove Theorem 1, it suffices to show that

$$\lim_{T\to\infty}\frac{K_T}{\sum_{i=1}^{K_T}\tau_i}=0,\quad a.s. \tag{12}$$

In the following, we will first prove that $K_T\to\infty$ almost surely as $T\to\infty$, and then show (12) holds.

Consider a "random walk" $\{\Omega_k\}_{k=0}^{\infty}$, which starts with 1 and increments with $A-1$. Denote the first 0-hitting time for $\{\Omega_k\}_{k=0}^{\infty}$ as κ. Then, $\Omega_0=1$, $\Omega_\kappa=0$. Define a random process $\{\exp(-\alpha\Omega_k-\gamma(\alpha)k)\}_{k=0}^{\infty}$ with $\alpha>0$ and $\gamma(\alpha)=e^{-\alpha}-(1-\alpha)>0$. We note that

$$\begin{aligned}
&\mathbb{E}\{\exp[-\alpha\Omega_k-\gamma(\alpha)k]|\exp(-\alpha\Omega_0),\ldots,\\
&\quad\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]\}\\
&=\mathbb{E}\{\exp[-\alpha(\Omega_{k-1}+A-1)-\gamma(\alpha)(k-1+1)]|\\
&\quad\exp(-\alpha\Omega_0),\ldots,\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]\}\\
&=\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]\exp[\alpha-\gamma(\alpha)]\mathbb{E}\{\exp(-\alpha A)|\\
&\quad\exp(-\alpha\Omega_0),\ldots,\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]\}\\
&=\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]
\end{aligned}$$

where the last equality follows from the assumption that A is a Poisson random variable with parameter 1 and is independent with the random walk prior to time slot k. Thus, it is a Martingale process. Based on the property of a Martingale, we have

$$\begin{aligned}
&\mathbb{E}\{\exp[-\alpha\Omega_k-\gamma(\alpha)k]\}\\
&=\mathbb{E}\{\mathbb{E}\{\exp[-\alpha\Omega_k-\gamma(\alpha)k]|\exp(-\alpha\Omega_0),\ldots,\\
&\qquad\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]\}\}\\
&=\mathbb{E}\{\exp[-\alpha\Omega_{k-1}-\gamma(\alpha)(k-1)]\}
\end{aligned}$$

Applying this equality recursively, we have

$$\begin{aligned}\exp(-\alpha\Omega_0) &= \mathbb{E}\{\exp[-\alpha\Omega_\kappa - \gamma(\alpha)\kappa]\} &(13)\\ &= \mathbb{E}\{(\mathbf{1}_{\kappa<\infty} + \mathbf{1}_{\kappa=\infty}) \cdot \exp[-\alpha\Omega_\kappa - \gamma(\alpha)\kappa]\}\\ &= \mathbb{E}\left[\mathbf{1}_{\kappa<\infty} \cdot \exp(-\alpha\Omega_\kappa - \gamma(\alpha)\kappa)\right] &(14)\end{aligned}$$

where the equality in (14) holds due to the fact that $\exp[-\gamma(\alpha)\cdot\infty] = 0$. Let $\alpha \to 0^+$, then $\gamma(\alpha) \to 0^+$, and the equation becomes

$$1 = \mathbb{E}[\mathbf{1}_{\kappa<\infty}] = \mathbb{P}[\kappa < \infty] \tag{15}$$

i.e., the probability of hitting 0 in finite time is 1.

We point out that (14) holds for any initial state Ω_0, so does (15). Thus, starting with any $e_i > 0$, the probability that the first 0-hitting time is finite equals 1, i.e., $\mathbb{P}[v_i < \infty] = 1$. This implies that for arbitrary time t, the battery will become empty within finite time after it with probability one. Thus, $\lim_{T\to\infty} \mathbb{P}[K_T < \infty] = 0$, i.e., $K_T \to \infty$ almost surely as $T \to \infty$.

Since $\Omega_\kappa = 0$, (13) is equivalent to

$$\mathbb{E}\left[\exp(-\gamma(\alpha)\kappa)\right] = \exp(-\alpha).$$

We note that by shifting Γ_i to initial time index 1, it virtually follows the same random walk $\{\Omega_k\}_k$. For such K_T i.i.d random walks with 0-hitting times τ_i, we have

$$\mathbb{E}\left[\exp\left(-\gamma(\alpha)\left(\sum_{i=1}^{K_T}\tau_i\right)\right)\right] = \exp(-K_T\alpha). \tag{16}$$

Therefore,

$$\begin{aligned}&\mathbb{P}\left[\frac{K_T}{\sum_{i=1}^{K_T}\tau_i} > \epsilon\right] = \mathbb{P}\left[\sum_{i=1}^{K_T}\tau_i < \frac{K_T}{\epsilon}\right]\\ &= \mathbb{P}\left[\exp\left(-\gamma(\alpha)\left(\sum_{i=1}^{K_T}\tau_i\right)\right) > \exp\left(-\gamma(\alpha)\frac{K_T}{\epsilon}\right)\right] &(17)\\ &\le \frac{\exp(-K_T\alpha)}{\exp(-\gamma(\alpha)\frac{K_T}{\epsilon})} = \exp\left(-K_T\left(\alpha - \frac{\gamma(\alpha)}{\epsilon}\right)\right) &(18)\end{aligned}$$

where (17) follows from the monotonicity of e^{-x} and (18) follows from Markov's inequality and (16). Since $\gamma(\alpha) = O(\alpha^2)$, for any $\epsilon > 0$, we can always find an α to have $\alpha - \frac{\gamma(\alpha)}{\epsilon} > 0$, and then the probability decays exponentially in K_T. This implies that

$$\sum_{K_T=1}^{\infty} \mathbb{P}\left[\frac{K_T}{\sum_{i=1}^{K_T}\tau_i} > \epsilon\right] < \infty.$$

According to Borel-Cantelli lemma [15], if the sum of the probabilities of a sequence of events is finite, then the probability that infinitely many of them occur is 0. Therefore,

$$\mathbb{P}\left(\limsup_{n\to\infty} \frac{K_T}{\sum_{i=1}^{K_T}\tau_i} > \epsilon\right) = 0,$$

which implies (12). This completes the proof.

B. Proof of Theorem 2

To prove Theorem 2, it suffices to show that

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n) \le f(1),\ a.s.$$

As illustrated in Fig. 6, there are v_i equally spaced sensing epochs in the segment corresponding to v_i. Considering the duration bounded by the first and last sensing epochs in the segment, the aggregated estimation MSE equals $(v_i - 1)f(1)$. The duration bounded by the last sensing epoch in the segment associated with v_i and the first sensing epoch in the segment associated with v_{i+1} is $f(u_i + 1)$. Therefore,

$$\begin{aligned}&\limsup_{T\to+\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\\ &= \limsup_{T\to+\infty} \frac{f(u_0) + \sum_{i=1}^{K_T}\left[(v_i-1)f(1) + f(u_i+1)\right]}{T}\\ &= \limsup_{T\to+\infty} \frac{f(u_0) + \sum_{i=1}^{K_T} f(u_i+1)}{T}\\ &\quad + \frac{T - \sum_{i=0}^{K_T} u_i - K_T}{T} f(1) &(19)\\ &\le \limsup_{T\to+\infty} f(1) - \frac{\sum_{i=0}^{K_T} u_i}{T} f(1) - \frac{K_T}{T} f(1)\\ &\quad + \frac{\sum_{i=0}^{K_T} C u_i}{T} + \frac{K_T C}{T}\\ &= f(1) \qquad a.s. &(20)\end{aligned}$$

where (19) follows from the fact that $u_0 + \sum_{i=1}^{K_T}(v_i + u_i) = T$, and (20) follows from Assumptions 1-3) and the fact that $K_T/T \to 0$ and $\sum_{i=0}^{K_T} u_i/T \to 0$ almost surely, as proved in the proof of Theorem 1.

Since

$$\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n) \le \frac{1}{T}\left(\sum_{n=1}^{N_T+1} C d_n\right) = C,$$

it is uniformly bounded in T. By the Bounded Convergence Theorem [16], we have

$$\begin{aligned}\limsup_{T\to\infty} \mathbb{E}\left(\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right) &= \mathbb{E}\left(\limsup_{T\to\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right)\\ &= f(1).\end{aligned}$$

C. A Virtual Energy Harvesting Sensing System

Before we define the virtual sensing system in this section, we first introduce the following Lemma, which will be used later to characterize the virtual battery evolution process.

Lemma 3: Consider a Poisson random variable A with parameter λ. Given $A \ge x$ for some positive integer x, we have $x < \mathbb{E}[A|A \ge x] < x + \lambda$.

Proof: Define B as a random variable with PMF

$$\mathbb{P}[B = i] = \frac{\mathbb{P}[A = x + i]}{\mathbb{P}[A \geq x]}, \quad i = 0, 1, 2, \ldots$$

Then,

$$\mathbb{E}[A|A \geq x] = \frac{\sum_{i=0}^{\infty} \mathbb{P}[A = x + i](x + i)}{\mathbb{P}[A \geq x]} \tag{21}$$

$$= \sum_{i=0}^{\infty} \mathbb{P}[B = i](x + i) = x + \mathbb{E}[B] \tag{22}$$

$$= x + \sum_{n=0}^{\infty} \mathbb{P}[B > n] > x \tag{23}$$

Thus, in order to prove the other inequality in Lemma 3, it suffices to prove that $\mathbb{P}[B > n] < \mathbb{P}[A > n]$ for $n = 0, 1, 2, \ldots$, which is equivalent to $\mathbb{P}[B \leq n] > \mathbb{P}[A \leq n]$ for $n = 0, 1, 2, \ldots$. Based on the definition of A and B, it then suffices to show that

$$\frac{\sum_{i=0}^{n} \lambda^{x+i}/(x + i)!}{\sum_{j=0}^{\infty} \lambda^{x+j}/(x + j)!} > \frac{\sum_{i=0}^{n} \lambda^{i}/i!}{\sum_{j=0}^{\infty} \lambda^{j}/j!} \tag{24}$$

i.e.,

$$\sum_{i=0}^{n}\sum_{j=0}^{\infty} \frac{\lambda^{x+i+j}}{(x + i)!j!} > \sum_{i=0}^{n}\sum_{j=0}^{\infty} \frac{\lambda^{x+i+j}}{(x + j)!i!} \tag{25}$$

Since

$$\sum_{i=0}^{n}\sum_{j=0}^{n} \frac{\lambda^{x+i+j}}{(x + i)!j!} > \sum_{i=0}^{n}\sum_{j=0}^{n} \frac{\lambda^{x+i+j}}{(x + j)!i!}, \tag{26}$$

it then suffices to show that for $i = 0, 1, \ldots, n$, $j = n + 1, n + 2, \ldots$, $\frac{1}{(x+i)!j!} > \frac{1}{(x+j)!i!}$. This is true since $j > i$, $x > 0$. ■

Consider an energy harvesting sensing system with a virtual battery whose state can be any integer in $(-\infty, +\infty)$. It senses every $\frac{1}{1-\beta}$ units of time, even if the battery state is zero or negative. The energy arrives at the virtual battery according to a Poisson process with parameter 1. Each sensing operation consumes one unit of energy. We use $E_\beta(n)$ to denote the battery state right before the n-th sensing epoch, i.e., at time $\frac{n}{1-\beta}$. Assume the system starts with initial energy level x, then, the battery status evolves according to

$$E_\beta(0) = x \tag{27}$$

$$E_\beta(n) = E_\beta(n - 1) + A\left(\frac{1}{1 - \beta}\right) - 1, \quad n = 1, 2, \ldots \tag{28}$$

where $A\left(\frac{1}{1-\beta}\right)$ is a Poisson random variable with parameter $\frac{1}{1-\beta}$. Thus,

$$\mathbb{E}[E_\beta(n)] = x + \frac{\beta}{1 - \beta}n \tag{29}$$

Therefore, when $0 < \beta < 1$, the energy level drifts up in expectation; Otherwise, when $\beta < 0$, it drifts down.

Define

$$\Lambda_\beta(\alpha) := \log \mathbb{E}\left[e^{-\alpha\left(A\left(\frac{1}{1-\beta}\right)-1\right)}\right] = \frac{e^{-\alpha} - 1}{1 - \beta} + \alpha \tag{30}$$

We note that $\Lambda_\beta(\alpha)$ is convex in α, $\Lambda_\beta(0) = 0$, and $\Lambda'_\beta(\alpha) = -\frac{e^{-\alpha}}{1-\beta} + 1$. Thus, equation $\Lambda_\beta(\alpha) = 0$ has another root besides 0, denoted as α_0. We have

$$\frac{e^{-\alpha_0} - 1}{1 - \beta} + \alpha_0 = 0, \quad \Lambda'_\beta(0) = -\frac{\beta}{1 - \beta} \tag{31}$$

When α_0 is sufficiently small, we have

$$\beta = \frac{\alpha_0}{2} + o(\alpha_0) \tag{32}$$

Assume $x \in (0, M)$, where M is a positive integer. We are interested in the event that the random process $\{E_\beta(n)\}_{n=0}^{\infty}$ hits or exceeds one of the two boundary levels 0 and M for the first time. We have the following observations.

Lemma 4: Consider the random process $\{E_\beta(n)\}_{n=0}^{\infty}$ defined in (27)-(28). Letting κ be the smallest n such that $E_\beta(n) \geq M$ or $E_\beta(n) = 0$, and $\tau_x := \mathbb{E}[\kappa]$. Define $P_{x,M}$ as the probability that $E_\beta(\kappa) \geq M$, and $P_{x,0}$ as the probability that $E_\beta(\kappa) = 0$. Then,

$$P_{x,M} = \frac{1 - e^{-\alpha_0 x}}{1 - e^{-\alpha_0(M+\theta_x)}} \tag{33}$$

$$P_{x,0} = \frac{e^{-\alpha_0 x} - e^{-\alpha_0(M+\theta_x)}}{1 - e^{-\alpha_0(M+\theta_x)}} \tag{34}$$

$$\tau_x = \frac{1 - \beta}{\beta}((M + \phi_x)P_{x,M} - x) \tag{35}$$

where $0 \leq \theta_x \leq \frac{1}{1-\beta}$, $0 \leq \phi_x \leq \frac{1}{1-\beta}$.

Proof: Define $\Omega_n := \exp(-\alpha(E_\beta(n) + \Lambda_\beta(\alpha)n))$. Then, $\{\Omega_n\}_{n=0}^{\infty}$ is a martingale process with initial state $\Omega_0 = \exp\{-\alpha x\}$. Based on the definition, we have

$$\begin{aligned}\mathbb{E}[\Omega_n] &= \mathbb{E}[\mathbb{E}[\Omega_n|\Omega_0, \ldots, \Omega_{n-1}]] = \mathbb{E}[\Omega_{n-1}] \\ &= \ldots = \mathbb{E}[\Omega_0] = \exp(-\alpha x)\end{aligned} \tag{36}$$

Taking derivative of both sides with respect to α, we have

$$\mathbb{E}[(E_\beta(n) + \Lambda'_\beta(\alpha)n)\Omega_n] = x \exp(-\alpha x) \tag{37}$$

Letting $\alpha \to 0$ in (36) for $n = \kappa$, we have

$$\begin{aligned}\text{LHS} &= \mathbb{E}[\Omega_\kappa] \\ &= \mathbb{E}[\Omega_\kappa|\text{first hits } M]P_{x,M} + \mathbb{E}[\Omega_\kappa|\text{first hits } 0]P_{x,0} \\ &= P_{x,M} + P_{x,0} = 1 = \text{RHS}\end{aligned} \tag{38}$$

Similarly, letting $\alpha \to \alpha_0$ in (36) for $n = \kappa$, we have

$$\begin{aligned}\text{LHS} &= \mathbb{E}[\Omega_\kappa|\text{first hits } M]P_{x,M} + P_{x,0} \\ &= \exp(-\alpha_0 x) = \text{RHS}\end{aligned} \tag{39}$$

We note

$$\begin{aligned}&\mathbb{E}[\Omega_\kappa|\text{first hits } M] \\ &= \mathbb{E}[\exp(-\alpha_0(E_\beta(\kappa) + \Lambda_\beta(\alpha_0)\kappa))|E_\beta(\kappa) \geq M] \\ &= \mathbb{E}[\exp(-\alpha_0 E_\beta(\kappa))|E_\beta(\kappa) \geq M] \\ &\leq e^{-\alpha_0 M}\end{aligned} \tag{40}$$

On the other hand, we have

$$\mathbb{E}[\exp(-\alpha_0 E_\beta(\kappa))|E_\beta(\kappa) \geq M] \geq \exp(-\alpha_0 \mathbb{E}[E_\beta(\kappa)|E_\beta(\kappa) \geq M]) \quad (41)$$
$$\geq e^{-\alpha_0\left(M+\frac{1}{1-\beta}\right)} \quad (42)$$

where (41) follows from Jensen's inequality, and (42) follows from Lemma 3.

Combining (39), (40) and (42), we have

$$P_{x,M} e^{-\alpha_0(M+\theta_x)} + P_{x,0} = e^{-\alpha_0 x}, \quad (43)$$

where $0 \leq \theta_x \leq \frac{1}{1-\beta}$.

Solving (38) and (43), we obtain (33)-(34).

Letting $\alpha \to 0$ in (37) for $n = \kappa$, we have

$$\begin{aligned}\text{LHS} &= \mathbb{E}[(E_\beta(\kappa) + \Lambda'_\beta(\alpha)\kappa)\exp(-\alpha)] \\ &= \mathbb{E}\left[E_\beta(\kappa) - \left(\frac{1}{1-\beta} - 1\right)\kappa\right] \\ &= (M + \phi_x)P_{x,M} - \frac{\beta}{1-\beta}\tau_x = x = \text{RHS}\end{aligned}$$

where $0 \leq \phi_x \leq \frac{1}{1-\beta}$. Thus, we have (35) established. ■

Lemma 5: Consider the random process $\{E_\beta(n)\}_{n=0}^\infty$ defined in (27)-(28). Define $S^-_{x,M}$ as the expected time index n when $\{E_\beta(n)\}_{n=0}^\infty$ with $\alpha_0 = -\frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) < 0$ hits boundary level M for the first time, and $S^+_{x,0}$ as the expected time index n when $\{E_\beta(n)\}_{n=0}^\infty$ with $\alpha_0 = \frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) > 0$ hits boundary level M for the first time. Then, $S^-_{M,M} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$, $S^+_{0,0} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$.

Proof: First, let us consider the case when $\alpha_0 = -\frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) < 0$. We use superscript $-$ to indicate that α_0 involved in the corresponding quantities is negative.

Applying Lemma 4 for $x = 1$ and $x = M-1$, we have

$$\begin{aligned}P^-_{1,M} &= \frac{1 - e^{-\alpha_0}}{1 - e^{-\alpha_0(M+\theta^-_1)}} = \frac{\alpha_0(1 + O(\alpha_0))}{-M^k(1 + O(\alpha_0 + M^{-k})} \\ &= \frac{\alpha_0}{-M^k}(1 + O(\alpha_0 + M^{-k}))\end{aligned}$$

$$\begin{aligned}P^-_{M-1,0} &= \frac{e^{-\alpha_0(M-1)} - e^{-\alpha_0(M+\theta^-_{M-1})}}{1 - e^{-\alpha_0(M+\theta^-_{M-1})}} \\ &= \frac{e^{\alpha_0} - e^{-\alpha_0\theta^-_{M-1}}}{e^{\alpha_0 M} - e^{-\alpha_0\theta^-_{M-1}}} \\ &= \frac{\alpha_0(1 + \theta^-_{M-1}(1 + O(\alpha_0))}{-1 + O(\alpha_0 + M^{-k})} \\ &= -\alpha_0(1 + \theta^-_{M-1})(1 + O(\alpha_0 + M^{-k}))\end{aligned}$$

For the corresponding expected first hitting time, we have

$$\begin{aligned}\tau^-_1 &= \frac{1-\beta}{\beta}\left(\left(M + \phi^-_1\right)P^-_{1,M} - 1\right) \\ &= \frac{1-\beta}{\beta}(-1 + o(1)) \\ &= -\frac{2}{\alpha_0}(1 + o(1))\end{aligned} \quad (44)$$

and

$$\begin{aligned}\tau^-_{M-1} &= \frac{1-\beta}{\beta}\left(\left(M + \phi^-_{M-1}\right)P^-_{M-1,M} - (M-1)\right) \\ &= \frac{1-\beta}{\beta}\left[\left(M + \phi^+_{M-1}\right)(1 - M^{-k}2\alpha_0(1 + o(1)) \right. \\ &\quad \left. -(M-1)\right] \\ &= \frac{1-\beta}{\beta}(\phi^-_{M-1} + 1)\alpha_0(1 + o(1)) \\ &= 2(M + \phi^-_{M-1})(1 + o(1))\end{aligned} \quad (45)$$

We note that

$$S^-_{M-1,M} = \tau^-_{M-1} + P^-_{M-1,0} \cdot S^-_{0,M} \quad (46)$$

$$S^-_{0,M} \geq \sum_{x=0}^{M} q_{0,x}\left(\tau_x + P^-_{x,0}S^-_{0,M}\right) \quad (47)$$

where $q_{0,x}$ is the probability that given the random process $\{E_\beta(n)\}_{n=0}^\infty$ first hits boundary 0, it re-enters the range $[0, M]$ with state x. Thus,

$$S^-_{0,M} \geq \frac{\sum_{x=0}^M q_{0,x}\tau_x}{1 - \sum_{x=0}^M q_{0,x}P^-_{x,0}} = \frac{\sum_{x=0}^M q_{0,x}\tau_x}{\sum_{x=0}^M q_{0,x}P^-_{x,M}} \quad (48)$$

According to (33), when $\alpha_0 x$ is sufficiently small, we have

$$P^-_{x,M} = \frac{1 - e^{-\alpha_0 x}}{1 - e^{-\alpha_0(M+\theta_x)}} = \frac{x\alpha_0}{-M^k}(1 + O(\alpha_0 + M^{-k})) \quad (49)$$

Pick the smallest positive integer K such that $\frac{1}{K!} < \frac{1}{M^{k+2}}$. Hence $K = O(\log M)$ and $\alpha_0 K = o(1)$. For sufficiently large M, $P^-_{x,M} \leq P^-_{K,M}$. Thus, we have

$$\begin{aligned}\sum_{x=0}^M q_{0,x}P^-_{x,M} &\leq \sum_{x=0}^K q_{0,x}P^-_{x,M} + \sum_{x=K+1}^M q_{0,x} \\ &\leq \left(\sum_{x=0}^K q_{0,x}\right)P^-_{K,M} + \sum_{x=K+1}^M q_{0,x} \\ &= (1-q)P^-_{K,M} + q\end{aligned}$$

where $q := \sum_{x=K+1}^M q_{0,x}$. By induction, we can show that $q = O\left(\frac{1}{(K-1)!}\right)$. Therefore,

$$S^-_{0,M} \geq \frac{q_{0,1}\tau^-_1}{P^-_{K,M}(1 + O(\alpha_0 + M^{-k}))} \quad (50)$$

Plugging (50) in (46), we have

$$\begin{aligned}S^-_{M-1,M} &\geq \tau^-_{M-1} + \frac{P^-_{M-1,0}q_{0,1}\tau^-_1}{P^-_{K,M}(1 + O(\alpha_0 + M^{-k}))} \quad (51) \\ &\geq 2M + \frac{M^k}{K}q_{0,1}\frac{2M}{k\log M}(1 + O(\alpha_0 + M^{-k})) \quad (52) \\ &\sim \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right) \quad (53)\end{aligned}$$

Since $S^-_{M-1,M} > (1 + S^-_{M-1,M})\mathbb{P}\left[A\left(\frac{1}{1-\beta}\right) = 0\right]$, we have $S^-_{M,M} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$.

Next, let us consider the case when $\alpha_0 = \frac{k \log M}{M} + o\left(\frac{\log M}{M}\right) > 0$. In the following, we use superscript + to indicate that α_0 involved in the corresponding quantities is positive.

Applying Lemma 4 for $x = 1$ and $x = M - 1$, we have

$$\begin{aligned} P^+_{1,M} &= \frac{1 - e^{-\alpha_0}}{1 - e^{-\alpha_0(M+\theta_1^+)}} = \frac{\alpha_0(1 + O(\alpha_0))}{1 + O(M^{-k})} \\ &= \alpha_0(1 + O(\alpha_0 + M^{-k})) \end{aligned}$$

and

$$\begin{aligned} P^+_{M-1,0} &= \frac{e^{-\alpha_0(M-1)} - e^{-\alpha_0(M+\theta^+_{M-1})}}{1 - e^{-\alpha_0(M+\theta^+_{M-1})}} \\ &= \frac{e^{-\alpha_0 M}(e^{\alpha_0} - e^{-\alpha_0\theta^+_{M-1}})}{1 - e^{-\alpha_0(M+\theta^+_{M-1})}} \\ &= \frac{M^{-k}\alpha_0(1 + \theta^+_{M-1} + O(\alpha_0))}{1 + O(M^{-k})} \\ &\leq M^{-k} \cdot 2\alpha_0(1 + O(\alpha_0 + M^{-k})) \end{aligned}$$

where the inequality follows from the fact that $\theta^+_{M-1} \leq \frac{1}{1-\beta} = 1 + O(\alpha_0)$. Thus,

$$\frac{P^+_{1,M}}{P^+_{M-1,0}} \geq \frac{M^k}{2}(1 + O(\alpha_0 + M^{-k})) \tag{54}$$

For the corresponding expected first hitting time, we have

$$\begin{aligned} \tau_1^+ &= \frac{1-\beta}{\beta}\left(\left(M + \phi_1^+\right) P^+_{1,M} - 1\right) \\ &= \frac{1-\beta}{\beta}\left(\left(M + \phi_1^+\right)\alpha_0(1 + o(1)) - 1\right) \\ &= 2\left(M + \phi_1^+\right)(1 + o(1)) \end{aligned} \tag{55}$$

and

$$\begin{aligned} \tau^+_{M-1} &= \frac{1-\beta}{\beta}\left(\left(M + \phi^+_{M-1}\right) P^+_{M-1,M} - (M-1)\right) \\ &= \frac{1-\beta}{\beta}\left(\left(M + \phi^+_{M-1}\right)(1 - M^{-k}2\alpha_0(1 + o(1)) \right. \\ &\quad \left. -(M-1)\right) \qquad (56) \\ &= \frac{1-\beta}{\beta}(\phi^+_{M-1} + 1)(1 + o(1)) \\ &= \frac{2(1 + \phi^+_{M-1})}{\alpha_0}(1 + o(1)) \qquad (57) \end{aligned}$$

Following similar arguments as in (46)-(50), we have

$$\begin{aligned} S^+_{1,0} &\geq \tau_1^+ + \frac{P^+_{1,M}}{P^+_{M-1,0}} \cdot \tau^+_{M-1} \\ &\geq 2M + \frac{M^k}{2}(1 + O(\alpha_0 + M^{-k})) \cdot \frac{2M}{k \log M} \qquad (58) \\ &\sim \frac{M^{k+1}}{k \log M} \qquad (59) \end{aligned}$$

where (58) follows from (54), (55) and (57). ■

D. Proof of Theorem 3

Now consider the energy state evolution process $\{E(s_n^-)\}_{n=1}^\infty$ under the proposed adaptive sensing scheduling policy. We focus on the portion of the random process lying in ranges $[0, B/2)$ and $(B/2, B]$, respectively. Comparing the random process $\{E(s_n^-)\}_{n=1}^\infty$ with the virtual battery evolution process defined in (27)-(28), we note that each portion can be treated as part of $\{E_\beta(n)\}_{n=0}^\infty$ lying in the corresponding range. Therefore, the characterization of $\{E_\beta(n)\}_{n=0}^\infty$ in Lemma 4 and Lemma 5 can be slightly modified to characterize $\{E(s_n^-)\}_{n=1}^\infty$.

Specifically, for the portion lying in $[0, B/2)$, we let $M = B/2$, $\beta = \frac{k \log B}{B} > 0$, then, the expected number of epochs between two consecutive battery outage events, i.e., $E(s_n^-) = 0$, can be bounded below by $S^+_{0,0}$. Thus, based on law of large numbers, the probability that a sensing epoch is infeasible is bounded above by $1/S^+_{0,0}$. Therefore, it scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$.

Similarly, for the portion lying in $[B/2, B)$, we map $B \to M$, $B/2 \to 0$, $\beta = -\frac{k \log B}{B} < 0$, then, the expected number of epochs between two consecutive battery overflow events, i.e., $E(s_n^-) = B$, can be bounded below by $S^-_{M,M}$. Again, based on law of large numbers, the rate of battery overflow scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$. Due to the properties of Poisson process, we can show that the amount of wasted energy per unit time is bounded by twice of the battery overflow rate, thus it scales in the same order.

E. Proof of Theorem 4

Consider the first n scheduled sensing epochs under the proposed adaptive sensing scheduling policy. Let n_+ denote the number of intervals between two scheduled sensing epochs with duration $\frac{1}{1-\beta}$, n_- be that with duration $\frac{1}{1+\beta}$, and n_0 be that with duration 1. Let $\bar{n}$ be the number of sensing epochs the battery overflows, and $\underline{n}$ be the number of infeasible sensing epochs. Then, the n-th scheduled sensing epoch happens at time $T_n := \frac{n_+}{1-\beta} + n_0 + \frac{n_-}{1+\beta}$. Let A_n^+ be the total amount of energy wasted. Then,

$$E(S_n^-) = (A(T_n) - A_n^+) - (n - \underline{n}) \tag{60}$$

where $A(T_n)$ is a Poisson random variable with parameter T_n. Dividing both sides by n and taking the limit as n goes to $+\infty$, we have

$$\lim_{n\to\infty} \frac{E(n)}{n} = \lim_{n\to\infty} \frac{A(T_n)}{T_n} \cdot \frac{T_n}{n} - \lim_{n\to\infty} \frac{A_n^+}{n} - \left(1 - \lim_{n\to\infty} \frac{\underline{n}}{n}\right)$$

Therefore,

$$\lim_{n\to\infty} \frac{T_n}{n} = 1 + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{61}$$

Based on Taylor expansion and (61), we have

$$\begin{aligned} &\lim_{n\to\infty} \frac{n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)}{T_n} \\ &\quad = f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right) \end{aligned}$$

On the other hand, due to the existence of infeasible sensing epochs, we have

$$\lim_{n\to\infty} \frac{\sum_n f(d_n) - \left[n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)\right]}{T_n}$$

$$\leq \lim_{n\to\infty} \frac{\sum_{d_n : d_n \geq \frac{1}{1-\beta}} f(d_n)}{T_n} \tag{62}$$

$$\leq \lim_{n\to\infty} \frac{\sum_{d_n : d_n \geq \frac{1}{1-\beta}} C d_n}{T_n} \tag{63}$$

$$\leq \lim_{n\to\infty} \frac{2C\underline{n}}{T_n} = O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{64}$$

where (62) follows from the fact that the difference between the actual sensing performance and scheduled sensing performance is due to the infeasible sensing epochs. (63) follows from the property of $f(d)$, and (64) follows from Theorem 3 and (61). Thus,

$$\lim_{n\to\infty} \frac{\sum_n f(d_n)}{T_n} = f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$$

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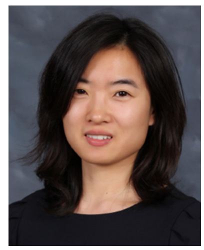

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